

Collecting like terms

1) Use shapes to identify terms from the same families.
Remember to include the sign when drawing shapes!

2) Add/subtract within families

Example:

Simplify $2x^2 - 2y + 3xy - 6x^2 + 5y - 4xy$

$-4x^2 + 3y - xy$

Index Laws: Algebraic

$$x^a \times x^b = x^{a+b}$$

$$x^a \div x^b = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

Example 1:

Simplify $2x^2y \times 4x^3y^4$

$$\begin{aligned} 2 \times 4 &= 8 \\ x^2 \times x^3 &= x^{2+3} = x^5 \\ y^1 \times y^4 &= y^{1+4} = y^5 \end{aligned}$$

$$8x^5y^5$$

Example 2:

Simplify $10x^3y^8 \div 2x^5y^2$

$$\begin{aligned} 10 \div 2 &= 5 \\ x^3 \div x^5 &= x^{3-5} = x^{-2} \\ y^8 \div y^2 &= y^{8-2} = y^6 \end{aligned}$$

$$5x^{-2}y^6$$

Example 3:

Simplify $(2x^4y)^3$

$$\begin{aligned} 2^3 &= 8 \\ (x^4)^3 &= x^{4 \times 3} = x^{12} \\ (y^1)^3 &= y^{1 \times 3} = y^3 \end{aligned}$$

$$8x^{12}y^3$$

	x	$+2$
$3x$	$3x^2$	$+6x$

$$\begin{array}{cc} x & +3 \\ \hline 5 & \begin{array}{|c|c|} \hline 5x & +15 \\ \hline \end{array} \end{array}$$

$$\begin{array}{cc} x & -4 \\ \hline -2 & \begin{array}{|c|c|} \hline -2x & +8 \\ \hline \end{array} \end{array}$$

$$= 5x + 15 - 2x + 8$$

$$3x + 23$$

$$= 3x + 23$$

Expanding double brackets

	$3x$	$+2$
x	$3x^2$	$+ 2x$
-4	$-12x$	-8

$$= 3x^2 + 2x - 12x - 8$$

$$= 3x^2 - 10x - 8$$

Expanding triple brackets

- 1) Expand first two brackets (using grid)
- 2) Use grid to multiply answer with third bracket

Example:

Expand and simplify $(x + 5)(2x - 1)(x - 3)$

	x	+5	
2x	$2x^2$	$+10x$	$2x^2 + 9x - 5$
-1	$-x$	-5	

	$2x^2$	$+9x$	-5	
x	$2x^3$	$+9x^2$	$-5x$	$2x^3 + 9x^2 - 6x^2 - 5x - 27x + 15$ $2x^3 + 3x^2 - 32x + 15$
-3	$-6x^2$	$-27x$	$+15$	

NOTE: Watch out for signs especially $-x - = +$

Factorising: linear expressions

1) Factor out: HCF (number) and Lowest Power (letter)

2) Fill in brackets: how much extra do you need

Example:

Factorise $25x^2y + 30x^5y^3$

$$5x^2y(5 + 6x^3y^2)$$

- HCF of 25 and 30 is 5
- Lowest power of x^2 and x^5 is x^2
- Lowest power of y and y^3 is y

Example (taking out a common linear factor):

Factorise and simplify $30(x + 1) - 10(x + 1)^2$

$$10(x + 1)(3 - 1(x + 1))$$

$$10(x + 1)(3 - x - 1)$$

$$10(x + 1)(2 - x)$$

- HCF of 30 and 10 is 10
- Lowest power of $(x+1)$ and $(x+1)^2$ is $(x+1)$

Factorising: quadratic expressions $a = 1$ 1) $a \times c$ 2) List factors of ac 3) Select pair which add/subtract to give b

4) Fill in grid

5) Factorise outside grid

6) Brackets

$ax^2 + bx + c$

+ c		- c	
+	+	+	-
-	-	-	+

Example 1:Factorise $x^2 + 4x - 12$

	x	-2
x	x^2	$-2x$
$+6$	$+6x$	-12

$a \times c = 1 \times -12$

$= -12$

1	12
2	6
3	4

 c is negative so signs are different...

$+2 - 6 = -4$

$-2 + 6 = +4 \checkmark$

ANSWER: $(x - 2)(x + 6)$ **Example 2:**Factorise $x^2 - 5x + 6$

	x	-2
x	x^2	$-2x$
-3	$-3x$	$+6$

$a \times c = 1 \times 6$

$= 6$

1	6
2	3

 c is positive so both signs are the same...

$+2 + 3 = +5$

$-2 - 3 = -5 \checkmark$

ANSWER: $(x - 2)(x - 3)$

Factorising: quadratic expressions $a > 1$

- 1) $a \times c$
- 2) List factors of ac
- 3) Select pair which add/subtract to give b
- 4) Fill in grid
- 5) Factorise outside grid
- 6) Brackets

$$ax^2 + bx + c$$

+ c		- c	
+	+	+	-
-	-	-	+

Example 1:Factorise $2x^2 - 7x - 15$

	$2x$	3
x	$2x^2$	$3x$
-5	$-10x$	-15

$$a \times c = 2 \times -15$$

$$= -30$$

1	30
2	15
3	10
5	6

c is negative so signs are different...

$$-3 + 10 = 7$$

$$+3 - 10 = -7 \checkmark$$

ANSWER: $(2x + 3)(x - 5)$ **Example 2:**Factorise $12x^2 + 5x - 3$

	$3x$	-1
$4x$	$12x^2$	$-4x$
$+3$	$+9x$	-3

$$a \times c = 12 \times -3$$

$$= -36$$

1	36
2	18
3	12
4	9
6	6

c is negative so signs are different...

$$-4 + 9 = +5 \checkmark$$

$$+4 - 9 = -5$$

ANSWER: $(2x + 3)(x - 5)$

Need to take the HCF out as factor.

In this case $4x$ is the HCF of $12x^2 - 4x$ Factorising: DOTS

$$a^2 - b^2 = (a + b)(a - b)$$

Example 1:Factorise $x^2 - 9$

$$(x + 3)(x - 3)$$

Example 2:Factorise $81x^2 - y^2$

$$(9x + y)(9x - y)$$

Example 3:Factorise fully $75x^2 - 27$

1. Take out HCF

2. DOTS

$$3(25x^2 - 9)$$

$$3(5x + 3)(5x - 3)$$

Algebraic fractions: Simplifying

1) Factorise

2) Cancel

Example 1:

$$\begin{aligned} \text{Simplify: } & \frac{40x^2x}{32x} \\ & \frac{5x}{4} \end{aligned}$$

Example 2:

$$\begin{aligned} \text{Simplify: } & \frac{35x^2 + 20}{5} \\ & = \frac{5(7x + 4)}{5} \\ & = 7x + 4 \end{aligned}$$

Example 3:

$$\begin{aligned} \text{Simplify: } & \frac{(x-3)}{(x-4)(x-3)} \\ & = \frac{1}{x-4} \end{aligned}$$

Example 4:

$$\begin{aligned} \text{Simplify: } & \frac{x^2 + 10x + 24}{x^2 - 36} \quad \leftarrow \text{DOTS} \\ & = \frac{(x+6)(x+4)}{(x+6)(x-6)} \\ & = \frac{x+4}{x-6} \end{aligned}$$

Algebraic fractions: multiplying/dividing

- 1) Factorise numerators and denominators as much as possible
- 2) For divide: KEEP - CHANGE (to x) - FLIP
- 3) Cancel common factors in numerator & denominator

Example 1:

$$\begin{aligned} \text{Simplify } \frac{x^2 + 5x + 6}{4} \times \frac{2}{x+2} \\ &= \frac{(x+3)(x+2)}{4} \times \frac{2}{(x+2)} \\ &= \frac{\cancel{(x+3)}\cancel{(x+2)}}{2\cancel{(x+2)}} \\ &= \frac{x+3}{2} \end{aligned}$$

Example 2:

Show that $\frac{1}{6x^2 + 7x - 5} \div \frac{1}{4x^2 - 1}$ simplifies to $\frac{ax + b}{cx + d}$ where a, b, c and d are integers.

Grid DOTS

Factorise everything: $\frac{1}{(2x-1)(3x+5)} \div \frac{1}{(2x+1)(2x-1)}$

Keep-Change (to x)-Flip: $\frac{1}{(2x-1)(3x+5)} \times \frac{(2x+1)(2x-1)}{1}$

$$\frac{(2x+1)\cancel{(2x-1)}}{\cancel{(2x-1)}(3x+5)} = \frac{(2x+1)}{(3x+5)}$$

Algebraic fractions: Addition/subtraction

- 1) Cross multiply (including denominators!)
- 2) Write as one fraction
- 3) Expand and simplify

Example:

$$\begin{aligned} \text{Simplify } \frac{2x+1}{2} - \frac{x+3}{5} \\ &= \frac{5(2x+1) - 2(x+3)}{10} \quad \text{careful! } -2x+3 = -6 \\ &= \frac{10x+5-2x-6}{10} \\ &= \frac{8x-1}{10} \end{aligned}$$

Algebraic fractions: Addition/subtraction

1) Make lowest common denominator: include each term and/or bracket with its lowest power

2) Multiply numerators

3) Expand and simplify where necessary

Lowest Common Denominator Example:

Express $\frac{5}{(x+3)^2} + \frac{8}{x(x+3)}$ as a single fraction in its simplest form.

$$\begin{array}{l}
 \begin{array}{c} \text{xx} \end{array} \left(\frac{5(x)}{x(x+3)^2} + \frac{8(x+3)}{x(x+3)^2} \right) \begin{array}{c} \text{x(x+3)} \end{array} \\
 \frac{5x + 8(x+3)}{x(x+3)^2} \\
 \frac{5x + 8x + 24}{x(x+3)^2} \\
 \frac{13x + 24}{x(x+3)^2}
 \end{array}$$

Lowest Common Denominator: $x(x+3)^2$

Solving Equations: Linear

1) Kill, kill

2) Get x's on one side and numbers on the other

* x's on both sides: Kill smallest x first *

Example 1:

Solve $2x - 4 = 6$

$$+4 \quad +4$$

$$2x = 10$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

Example 2:

Solve $5x - 6 = 2x + 9$

$$-2x \quad -2x$$

$$3x - 6 = 9$$

$$+6 \quad +6$$

$$\frac{3x}{3} = \frac{15}{3}$$

$$x = 5$$

Solving Equations: Linear with fractions

1) Kill anything not trapped by fraction line

2) Kill fractions by multiplying

Example 1:

Solve $\frac{x}{3} - 2 = 5$ kill the -2 first as its not trapped by the fraction line

$$\begin{array}{r} +2 \quad +2 \\ \frac{x}{3} = 7 \\ \times 3 \quad \times 3 \\ x = 21 \end{array}$$

Example 2:

Solve $\frac{x+2}{4} = 3$ kill the 4 first as everything is trapped by the fraction line

$$\begin{array}{r} \times 4 \quad \times 4 \\ x+2 = 12 \\ -2 \quad -2 \\ x = 10 \end{array}$$

Example 3:

Solve $\frac{x-2}{4} = \frac{x+3}{5}$ cross multiply to kill both 4 and 5 (DO NOT MULTIPLY DENOMINATORS)

$$\begin{array}{r} 5(x-2) = 4(x+3) \\ 5x-10 = 4x+12 \\ -4x \quad -4x \\ x-10 = 12 \\ +10 \quad +10 \\ x = 22 \end{array}$$

Solving Equations: Quadratics by factorising

1) Rearrange = 0

2) Factorise

3) Put each bracket = 0 and solve

Example 1:

Solve $x^2 - 2x - 3 = 0$

$$(x-3)(x+1) = 0$$

$$\begin{array}{l} x-3=0 \quad x+1=0 \\ +3 \quad +3 \quad -1 \quad -1 \\ \hline x=3 \quad x=-1 \end{array}$$

Example 2:

Solve $x^2 + 5x = 0$

$$x(x+5) = 0$$

$$\begin{array}{l} x=0 \quad x+5=0 \\ \hline x=-5 \end{array}$$

Example 3:

Solve $4x + 2 = x^2 + 3x$

kill LHS to keep x^2 positive

$$\begin{array}{l} 4x + 2 = x^2 + 3x \\ -2 \quad -2 \\ \hline 4x = x^2 + 3x - 2 \\ -4x \quad -4x \\ \hline 0 = x^2 - x - 2 \\ (x-2)(x+1) = 0 \\ \begin{array}{l} x-2=0 \quad x+1=0 \\ +2 \quad +2 \quad -1 \quad -1 \\ \hline x=2 \quad x=-1 \end{array} \end{array}$$

Example 4:

Solve $\frac{4}{x+8} = \frac{x}{x+3}$

$$\begin{array}{l} 4(x+3) = x(x+8) \\ 4x+12 = x^2+8x \\ -4x \quad -4x \\ \hline 12 = x^2+4x \\ -12 \quad -12 \\ \hline 0 = x^2+4x-12 \\ (x+6)(x-2) = 0 \\ \begin{array}{l} x+6=0 \quad x-2=0 \\ -6 \quad -6 \quad +2 \quad +2 \\ \hline x=-6 \quad x=2 \end{array} \end{array}$$

Solving Equations: algebraic fractions

1) Cross multiply (including denominators)

2) Write as one fraction

3) Kill denominator by multiplying

4) Expand and simplify

5) Solve

Example 1:

Solve $\frac{x-3}{4} = \frac{x-8}{3}$

careful! $-4 \times -8 = +32$

$$\begin{array}{l} 3(x-3) = 4(x-8) \\ 3x-9 = 4x-32 \\ -3x \quad -3x \\ \hline -9 = x-32 \\ +32 \quad +32 \\ \hline x=23 \end{array}$$

Example 2:

Solve $\frac{4}{x+1} = \frac{2}{x-2}$

$$\begin{array}{l} 4(x-2) = 2(x+1) \\ 4x-8 = 2x+2 \\ -2x \quad -2x \\ \hline -8 = x+2 \\ -2 \quad -2 \\ \hline -6 = x \\ x=-6 \end{array}$$

Solving Equations: quadratic formula

To solve quadratic equations of the form $ax^2 + bx + c = 0$ use the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

sub a, b and c into:

$$x = \frac{-() \pm \sqrt{()^2 - 4()()}}{2()}$$

*On calculator: enter with a "+" first then use left button to go back and change to "-"

Example (calculator):

$$\text{Solve } 2x^2 - 3x - 10 = 0$$

Write your answers to 3sf.

$$a = 2 \quad b = -3 \quad c = -10$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-10)}}{2(2)}$$

$$\begin{array}{l} \swarrow + \quad \searrow - \\ x = 3.108495... \quad x = -1.608495... \\ x = 3.11 \quad x = -1.61 \end{array}$$

Example (non-calculator):

$$\text{Solve } x^2 - 6x - 8 = 0$$

Write your answer in the form $a \pm \sqrt{b}$ where a and b are integers.

$$a = 1 \quad b = -6 \quad c = -8$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 + 32}}{2}$$

$$x = \frac{6 \pm \sqrt{68}}{2}$$

$$x = \frac{6 \pm \sqrt{4 \times 17}}{2}$$

$$x = \frac{6 \pm 2\sqrt{17}}{2}$$

$$x = 3 \pm \sqrt{17}$$

Completing the square: $a = 1$

- 1) $(x \pm \quad)^2 - (\quad)^2$ For quadratics of the form $x^2 + bx + c$
- 2) Half coefficient of x
- 3) Subtract square
- 4) Bring number down
- 5) Simplify

Example:Express $x^2 - 6x + 11$ in the form $(x + a)^2 + b$

$$(x \quad)^2 - (\quad)^2 + 11$$

$$(x - 3)^2 - (-3)^2 + 11$$

$$(x - 3)^2 - 9 + 11$$

$$(x - 3)^2 + 2$$

$$\text{Answer: } (x - 3)^2 + 2$$

$$\text{where } a = -3 \text{ and } b = 2$$

Extension:State the turning point of $y = x^2 - 6x + 11$ Just simply remember $(-p, q)!$ $\longrightarrow y = (x - 3)^2 + 2$ Turning Point: $(3, 2)$ Where $(-p, q)$ comes from:1) x value: what makes bracket equal zero?

$$y = (x - 3)^2 + 2$$

$$x = 3$$

2) y value: when bracket is zero

$$y = (0)^2 + 2$$

$$y = 2$$

If $p = -3$
then $-p =$
3

Completing the square: $a > 1$

- 1) Take out factor of first two terms [$\quad$]
- 2) $(x \pm \quad)^2 - (\quad)^2$ For quadratics of the form $ax^2 + bx + c$
- 3) Half coefficient of x (when there is a coefficient of x^2)
- 4) Subtract square
- 5) Bring number down
- 6) Multiply out
- 7) Simplify

Example:Express $2x^2 + 20x - 7$ in the form $a(x + b)^2 + c$

$$2[x^2 + 10x] - 7$$

$$2[(x \quad)^2 - (\quad)^2] - 7$$

$$2[(x + 5)^2 - (5)^2] - 7$$

$$2[(x + 5)^2 - 25] - 7$$

$$2(x + 5)^2 - 50 - 7$$

$$2(x + 5)^2 - 57$$

$$\text{Answer: } 2(x + 5)^2 - 57$$

$$\text{where } a=2, b=5, c=-57$$

Extension:State the turning point of $y = 2x^2 + 20x - 7$ Just simply remember $(-p, q)!$ $\longrightarrow y = 2(x + 5)^2 - 57$ Turning Point: $(-5, -57)$

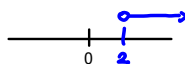
Linear inequalities

- 1) Kill, kill (same as solving equations)
- 2) If you $\times/\div$ by a negative number then flip inequality sign
- 3) On number line: $\circ$ for $<$ or $>$, $\bullet$ for $\leq$ or $\geq$

Example 1:

Solve the inequality and represent your solution on a number line

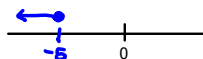
$$\begin{array}{l}
 5x + 7 > -x + 19 \\
 +x \quad +x \\
 6x + 7 > 19 \\
 -7 \quad -7 \\
 6x > 12 \\
 \frac{6x}{6} > \frac{12}{6} \\
 x > 2
 \end{array}$$



Example 2:

Solve the inequality and represent your solution on a number line

$$\begin{array}{l}
 12 - 3x \geq 27 \\
 -12 \quad -12 \\
 -3x \geq 15 \quad \star !! \\
 \frac{-3x}{-3} \geq \frac{15}{-3} \quad \text{divided by a negative number so flip inequality sign} \\
 x \leq -5
 \end{array}$$



Example 3:

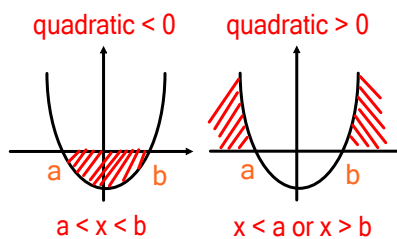
List the integers that satisfy the inequality

$$\begin{array}{l}
 -4 < 2n \leq 5 \\
 \frac{-4}{2} < \frac{2n}{2} \leq \frac{5}{2} \\
 -2 < n \leq 2.5
 \end{array}$$

answer: -1, 0, 1, 2

Quadratic inequalities

- 1) Rearrange to get all terms to one side (squared must be positive)
- 2) Factorise to find critical values
- 3) Sketch graph
- 4) Write inequality



Example 1:

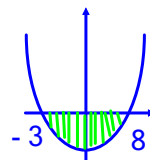
Solve $x^2 - 5x < 24$

$$x^2 - 5x - 24 < 0$$

Critical values: $x^2 - 5x - 24 = 0$

$$(x + 3)(x - 8) = 0$$

$$x = -3, x = 8$$



Answer: $-3 < x < 8$

Example 2:

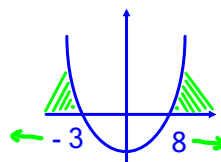
Solve $x^2 - 5x > 24$

$$x^2 - 5x - 24 > 0$$

Critical values: $x^2 - 5x - 24 = 0$

$$(x + 3)(x - 8) = 0$$

$$x = -3, x = 8$$



Answer: $x < -3, x > 8$

Index laws: Numerical

$$\begin{aligned}
 x^a \times x^b &= x^{a+b} & x^0 &= 1 \\
 x^a \div x^b &= x^{a-b} & x^{-a} &= \frac{1}{x^a} \quad \begin{array}{l} \text{1. Write as fraction} \\ \text{2. Flip} \end{array} \\
 (x^a)^b &= x^{ab} & \sqrt[b]{x^a} &= x^{\frac{a}{b}} \quad \begin{array}{l} \text{power} \\ \text{root} \end{array}
 \end{aligned}$$

Example 1:

Evaluate $3^{-2} = \left(\frac{1}{3}\right)^2 = \frac{1^2}{3^2} = \frac{1}{9}$

Example 2:

Evaluate $\left(\frac{125}{8}\right)^{-\frac{2}{3}}$

$$\begin{aligned}
 &= \left(\frac{8}{125}\right)^{\frac{2}{3}} \\
 &= \left(\frac{2^3}{5^3}\right)^{\frac{2}{3}} \\
 &= \frac{2^2}{5^2} \\
 &= \frac{4}{25}
 \end{aligned}$$

Sometimes you may need to solve equations which involve indices.

In this case, make the BASE numbers the same ...

Example 3:

Solve $2 \times 8^{2x} = 16^x$

$$2^1 \times (2^3)^{2x} = (2^4)^x \quad \text{change to base 2}$$

$$2^1 \times 2^{6x} = 2^{4x}$$

$$2^{1+6x} = 2^{4x}$$

equate powers

$$1 + 6x = 4x$$

$$2x = -1$$

$$x = \frac{-1}{2}$$

Ratio: using a ratio table

Example 1: Using a basic ratio table

Ali and Beth share £280 in the ratio 2:5.

Work out how much each person gets.

A	2	80
B	5	200
Total	7	280

$\times 40$ (from 2 to 80)
 $\times 40$ (from 7 to 280)

$\therefore$ Ali gets £80
 Beth gets £200

Example 2: Using a ratio table with MORE and LESS

Kim, Laura and Molly share £385

The ratio of the amount of money Kim gets to the amount of money Molly gets is 2:5

Kim gets £105 less than Molly gets.

How much does Laura get?

difference in parts = $5 - 2 = 3$
 $\div 3$ (3 parts = £105) $\div 3$ 1 part = £35

K	2	70
L		
M	5	175
Total		385

$385 - 70 - 175 = 140$
 $\therefore$ Laura gets £140

Ratio: using a ratio table

Example 3: Using a ratio table to ADD an amount

Riza has 35 red and yellow counters in a bag in the ratio 1:6.

She puts some more red counters into the bag.

The ratio of the number of red counters to the number of yellow counters is now 1:2.

How many red counters does Riza put into the bag?

R	1	5
Y	6	30
Total	7	35

$\times 5$ (from 1 to 5)
 $\times 5$ (from 7 to 35)

You add red counters but number of yellow stays the same.

R	1	15
Y	2	30
Total	3	

$\times 15$ (from 1 to 15)
 $\times 15$ (from 2 to 30)

$\therefore$ Riza puts 10 more red counters into the bag

Ratio: using fractions

Example 1: Using more than one ratio to find a fraction or percentage

On a school trip the ratio of the number of teachers to the number of students is 1:15

The ratio of the number of male students to the number of female students is 7:5

Work out what percentage of all the people on the trip are female students.
Give answer correct to nearest whole number.

represent ratios as fractions and use tree!

$$\begin{array}{l} T : S \\ \frac{1}{16} : \frac{15}{16} \end{array} \quad \begin{array}{l} MS : FS \\ \frac{7}{12} : \frac{5}{12} \end{array}$$

$$\begin{array}{l} \frac{1}{16} T \\ \swarrow \searrow \\ \frac{15}{16} S \end{array} \quad \begin{array}{l} \frac{7}{12} MS \\ \swarrow \searrow \\ \frac{5}{12} FS \end{array}$$

$$\frac{15}{16} \times \frac{5}{12} = \frac{25}{64}$$

$$\frac{25}{64} \times 100 = 39.0625$$

$$\underline{\underline{39\%}} \text{ (nearest whole number)}$$

Example 2: Using a diagram and fractions to find missing value

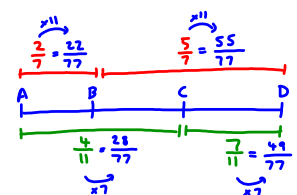
The points A, B, C and D lie in order on a straight line.

$$AB : BD = 2:5 \text{ and } AC : CD = 4:7$$

Find $AB : BC : CD$

Steps

1. draw diagram
2. label with fractions
3. find common denominator
4. find missing section
5. simplify ratio



$$\begin{aligned} BC \text{ is the missing section} &= AC - AB \\ &= \frac{28}{77} - \frac{22}{77} \\ &= \frac{6}{77} \end{aligned}$$

$$AB : BC : CD$$

$$\frac{22}{77} : \frac{6}{77} : \frac{49}{77} = \underline{\underline{22:6:49}}$$

Ratio: using ALGEBRAIC fractions

If $a : b = c : d$ then rewrite as $\frac{a}{b} = \frac{c}{d}$
then cross multiply and solve

Example 1: Solving algebraic ratios

Solve the following $x - 2 : x + 2 = x - 3 : x + 4$

$$\begin{aligned} \frac{x-2}{x+2} &= \frac{x-3}{x+4} \\ (x-2)(x+4) &= (x-3)(x+2) \\ x^2 + 2x - 8 &= x^2 - x - 6 \\ -x^2 & \quad -x^2 \\ 2x - 8 &= -x - 6 \\ +x & \quad +x \\ 3x - 8 &= -6 \\ +8 & \quad +8 \\ 3x &= 2 \\ x &= \frac{2}{3} \end{aligned}$$

Example 2: Solving algebraic ratios

$a = \sqrt{7} + \sqrt{c}$ and $b = \sqrt{63} + \sqrt{d}$ are c and d are positive integers.

Given that $c:d = 1:9$ find in its simplest form the ratio $a:b$

$$\begin{aligned} \textcircled{1} \quad \frac{c}{d} &= \frac{1}{9} \\ 9c &= d \\ \textcircled{2} \quad b &= \sqrt{63} + \sqrt{d} \\ &= 3\sqrt{7} + \sqrt{d} \\ &= 3\sqrt{7} + \sqrt{9c} \\ &= 3\sqrt{7} + 3\sqrt{c} \\ \textcircled{3} \quad \frac{a}{b} &= \frac{\sqrt{7} + \sqrt{c}}{3\sqrt{7} + 3\sqrt{c}} \\ &= \frac{1(\sqrt{7} + \sqrt{c})}{3(\sqrt{7} + \sqrt{c})} \\ \frac{a}{b} &= \frac{1}{3} \\ \therefore a:b &= \underline{\underline{1:3}} \end{aligned}$$

Ratio: using ALGEBRAIC fractions

Example 3: Solving equations for changing ratios

Steps

The ratio of the number of men to the number of women in an office is 2:3.

Four women leave the office.

The ratio of the number of men to the number of women in the office is now 4:5.

Work out the number of men in the office.

1. original ratio with x
2. expression for change
3. = new ratio
4. rewrite as fractions
5. cross multiply and solve

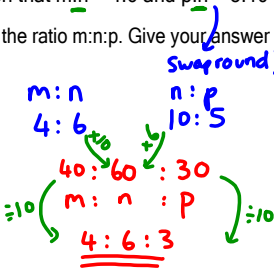
$$\begin{aligned} \textcircled{1} \quad M:W &= 2x:3x \\ \textcircled{2} \quad \text{Four women leave} &\rightarrow 2x:3x-4 \\ \textcircled{3} \quad 2x:3x-4 &= 4:5 \\ \textcircled{4} \quad \frac{2x}{3x-4} &= \frac{4}{5} \\ \textcircled{5} \quad 5(2x) &= 4(3x-4) \\ 10x &= 12x-16 \\ -10x & \quad -10x \\ 0 &= 2x-16 \\ +16 & \quad +16 \\ 16 &= 2x \\ \frac{16}{2} &= \frac{2x}{2} \\ x &= 8 \end{aligned}$$

Number of men in office originally was $2x$
men: $2x = 2(8) = 16$
 $\therefore 16$ men in the office

Ratio: combining ratios

Example 1:

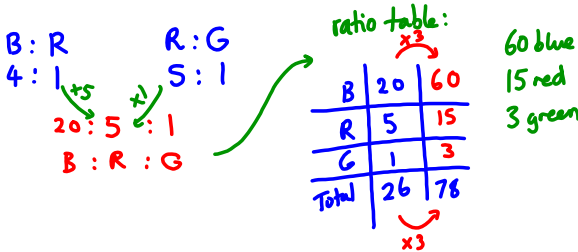
Given that $m:n = 4:6$ and $p:n = 5:10$
Find the ratio $m:n:p$. Give your answer in its simplest form.



Find common multiple for the matching part of the ratios

Example 2:

There are 4 times as many blue pens as red pens
and 5 times as many red pens as green pens in a box.
There are 78 pens in the box.
Find the number of pens for each colour.



TAGGED / SAMPLING

1) Let x = total number of ...

2) Use similar fractions to form an equation

3) Solve to find x

Example 1: 'Tagged' questions

Sarah wants to find out how many ducks there are in a park.

One day she puts a tag on each of the 30 ducks.

The next day she catches 20 ducks.

8 of these ducks have tags on them.

Work out an estimate for the number of ducks in the park.

$$\frac{\text{total}}{\text{tagged}} = \frac{\text{sample}}{\text{tagged sample}}$$

$$\frac{30}{x} = \frac{8}{20}$$

$$30(20) = 8(x)$$

$$600 = 8x$$

$$x = 75$$

Example 2: Stratified sample (where the sample is representative of the population)

Declan works in a confectioners. He is asked to test a sample of 40 chocolates stratified by type of chocolate. The table shows the number of each type of chocolate in the shop.

Type	Milk	Dark	White	Total
Number	600	220	130	950

Calculate the number of dark chocolates required for his stratified sample.

$$\frac{\text{total}}{\text{sample}} = \frac{\text{sample}}{\text{sample}}$$

$$\frac{220}{950} = \frac{x}{40}$$

$$220(40) = 950(x)$$

$$8800 = 950x$$

$$x = 9.2631...$$

9 dark chocolates are required for the sample

Example 3: Stratified sample (where the sample is representative of the population)

Hannah is planning a day trip for 195 students.

She asks a sample of 30 students where they want to go

Each student chooses one place.

The table shows information about her results

Place	Number of students
Theme Park	10
Theatre	5
Sports Centre	8
Seaside	7

(i) Work out how many of the 195 students you think will want to go to the Theme Park.

$$\frac{\text{Total}}{\text{sample}} = \frac{\text{sample}}{\text{sample}}$$

$$\frac{10}{30} = \frac{x}{195}$$

$$30(x) = 10(195)$$

$$30x = 1950$$

$$x = \frac{1950}{30}$$

$$x = 65$$

Exchange rates

1) Draw a proportion table and add exchange rate on top row

2) Add amount to change under correct currency

3) $x/\div$ as necessary

Example:

Harry finds out the price of a CD box set in three different countries.

The price is £98 in the UK, \$134.99 in the USA, 139.99 Euros in Germany.

The exchange rates are: $\text{£1} = \$1.43$ $1 \text{ Euro} = \text{£}0.73$

Harry wants to pay the cheapest price.

From which country should he buy the CD box set?

$$\begin{array}{c|c} \text{£1} & \$1.43 \\ \hline \text{£94.40} & \$134.99 \end{array}$$

$$\begin{array}{c|c} \text{£1} & \text{£}0.73 \\ \hline \text{£139.99} & \text{£}102.19 \end{array}$$

always convert to the currency that features in both exchange rates

UK: £98

USA: £94.40

Germany: £102.19

Therefore he should buy in USA

Direct and inverse proportion

Two quantities are in proportion if the ratio between the two quantities is constant.

1) Replace $\propto$ with $= k$

2) Sub in pair

3) Find k

4) Write general equation

• Direct Proportion:

$$y \propto x$$

$$y = kx$$

• Inverse Proportion:

$$y \propto 1/x$$

$$y = k/x$$

Example:

y is proportional to the square of x .

When $x = 3$, $y = 45$

a) Find a formula for y in terms of x

$$y \propto x^2$$

$$y = kx^2$$

$$45 = k(3)^2$$

$$\frac{45}{9} = \frac{9k}{9}$$

$$k = 5$$

$$\boxed{y = 5x^2}$$

b) Find y when $x =$

4

$$y = 5x^2$$

$$y = 5(4)^2$$

$$y = 5(16)$$

$$\boxed{y = 80}$$

Direct and inverse proportion (TABLE)

You can find a constant of proportionality if given a table of values for two quantities.

- 1) Rearrange equation to make k subject
- 2) Add additional rows to table
- 3) Find k values - if the same then it is a proportional relationship

Example:

Identify which of the following statements match to the tables of values below:

y	x	y	x^2
$y = k \times x$		$y = k \times x^2$	
$k = y \div x$		$k = y \div x^2$	

x^2	25	36
x	5	6
y	400	576
k	$400 \div 5 = 80$	$576 \div 6 = 96$

k	$400 \div 25 = 16$	$576 \div 36 = 16$	y	x^2
---	--------------------	--------------------	-----	-------

(where constant of proportionality is 16)

x^2	4	9
x	2	3
y	24	36
k	$24 \div 2 = 12$	$36 \div 3 = 12$

k	$24 \div 4 = 6$	$36 \div 9 = 4$	y	x
---	-----------------	-----------------	-----	-----

(where constant of proportionality is 12)

Machine days (proportion)

7 machines in a factory can produce 105 units in 5 days. U D
M If the machines all work at the same rate, how many machines M
 are needed to produce 156 units in 4 days? U D

Solve for K

$$U \propto MD$$

$$U = kMD$$

$$(105) = k(7)(5)$$

$$105 = k35$$

$$\underline{3 = k}$$

Find missing factor

$$U = 3MD$$

$$(156) = 3M(u)$$

$$156 = 12M$$

$$13 = M$$

1. Which factors affect the number of units?
2. $U = KMD$
3. Substitute
4. $M \times D$
5. Solve for K
6. Rewrite equation
7. Substitute
8. Solve for missing factor

Sequences: Linear

Linear sequences are expressed in the form $\text{---} n + \text{---}$

$\nearrow$
1st difference

$\nwarrow$
term before
1st term

Example:

a) Find the nth term rule for the following sequence:

$$\begin{array}{ccccccc}
 & +5 & & & & & \\
 & \text{---} & 8 & 11 & 14 & 17 & 20 \\
 & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \\
 & -3 & +3 & +3 & +3 & +3 &
 \end{array}$$

Answer: $3n + 5$

b) Find the 10th term of the sequence

$$\text{Let } n = 10$$

$$3(10) + 5 = 35$$

c) Is 48 a term in the sequence?

$$3n + 5 = 48$$

$$3n = 43$$

$$n = 14.333 \dots$$

Therefore 48 is not a term in the sequence.

To find if a term is in a sequence ...

1) make term = nth term rule

2) solve to find n

3) - YES if n is an integer

- NO if n is not an integer

Sequences: quadratic

- 1) Find 1st and 2nd differences
- 2) $a = 2\text{nd difference} \div 2$
- 3) Draw table
- 4) Find nth term for linear sequence (where necessary)

Example:

Find the nth term rule for the following sequence:

$4 \quad 13 \quad 26 \quad 43$
 $\quad \quad +9 \quad +13 \quad +17$
 $\quad \quad \quad +4 \quad +4$

$a = 4 \div 2 = 2$

n^2	1	4	9	16
Sequence	4	13	26	43
$2n^2$	2	8	18	32
subtract	2	5	8	11

$\Rightarrow 2n^2$
 $\Rightarrow +3n - 1$

Answer: $2n^2 + 3n - 1$

Simultaneous equations: linear

1) If necessary, multiply one or more equation to get x terms (or first letter) the same

2) Subtract the equations

3) Solve to find y

4) Sub answer back in to find x

Example 1:

Solve the following pair of simultaneous equations:

$$\begin{array}{rcl}
 2x + 2y = 14 & \rightarrow & 10x + 10y = 70 \\
 5x - 3y = 19 & \rightarrow & 10x + 6y = 38 \\
 \hline
 & & 16y = 32 \\
 & & 16 \quad 16 \\
 & & y = 2 \\
 2x + 2(2) = 14 & & \\
 2x + 4 = 14 & & \\
 -4 & -4 & \\
 \hline
 2x = 10 & & \\
 2 & 2 & \\
 \hline
 x = 5 & &
 \end{array}$$

touching signs!
+
-

Example 2:

Solve the following pair of simultaneous equations:

$$4x - 3y = 5 \text{ eqn. (1)}$$

$$x + y = -\frac{1}{2} \text{ eqn. (2)}$$

$$\begin{array}{rcl}
 4x - 3y = 5 & & \\
 4x + 4y = -2 & \text{eqn. (2) } \times 4 & \\
 \hline
 -7y = 7 & & \\
 y = -1 & &
 \end{array}$$

To find x:

$$4x - 3(-1) = 5 \text{ sub into eqn. (1)}$$

$$4x + 3 = 5$$

$$4x = 2$$

$$x = 0.5$$

Simultaneous equations: quadratic

- 1) Make x or y the subject of the linear equation
- 2) Sub in to quadratic equation
- 3) Rearrange to make it equal zero
- 4) Factorise and solve
- 5) Find corresponding values

Example:

Solve the following pair of simultaneous equations:

$$\begin{aligned} \textcircled{1} \quad x^2 + y^2 &= 13 \\ x - y &= 1 \\ \textcircled{2} \quad x &= 1 + y \end{aligned}$$

Make x the subject of the linear equation (easier to make x the subject here as y is negative)

Sub $\textcircled{2}$ into $\textcircled{1}$

$$(1+y)^2 + y^2 = 13$$

Sub linear equation into quadratic

$$1 + 2y + y^2 + y^2 = 13$$

$$2y^2 + 2y - 12 = 0$$

Rearrange to = 0

$$\frac{2y^2}{2} + \frac{2y}{2} - \frac{12}{2} = 0$$

$$y^2 + y - 6 = 0$$

Factorise and solve

$$(y+3)(y-2) = 0$$

$$y = -3 \quad y = 2$$

Sub in to $\textcircled{2}$

$$x = 1 + (-3) \quad x = 1 + 2$$

$$x = -2 \quad x = 3$$

Sub back into linear equation to find corresponding x values

Rearranging formula: subject appears once

- 1) Story of x
- 2) Inverse operations
- 3) Kill in reverse order

Example 1:

Make x the subject of:

$$y = \frac{x-4}{5}$$

Story of x:

$$1. -4 \quad 2. \div 5$$

$$5y = x - 4$$

$$5y + 4 = x$$

Example 2:

Make x the subject of:

$$y = \frac{x}{3} + 7$$

Story of x:

$$1. \div 3 \quad 2. + 7$$

$$y - 7 = \frac{x}{3}$$

$$3(y - 7) = x$$

$$3y - 21 = x$$

Example 3:

Make x the subject of:

$$y = (x-4)^2$$

Story of x:

$$1. -4 \quad 2. \square^2$$

$$\sqrt{y} = x - 4$$

$$\sqrt{y} + 4 = x$$

Example 4:

Make x the subject of:

$$y = 2x^3 - 8$$

Story of x:

$$1. \square^3 \quad 2. \times 2 \quad 3. - 8$$

$$\frac{y+8}{2} = x^3$$

$$\sqrt[3]{\frac{y+8}{2}} = x$$

Rearranging formula: subject appears twice

- 1) Multiply
- 2) Expand
- 3) Rearrange (get every term with subject on LHS and every term without subject on RHS)
- 4) Factorise
- 5) Divide

Example:

Make m the subject of:

$$f = \frac{4 - 3m}{5 + m}$$

$$\times (5+m) \quad \times (5+m)$$

$$f(5+m) = 4 - 3m$$

$$5f + fm = 4 - 3m$$

Kill 5f from LHS as it doesn't contain the subject (m) so we don't want it on LHS

$$-5f \quad -5f$$

$$fm = 4 - 3m - 5f$$

Kill -3m from RHS as it contains the subject (m) so we want it on LHS

$$+3m \quad +3m$$

$$fm + 3m = 4 - 5f$$

$$m(f+3) = 4 - 5f$$

$$\frac{m(f+3)}{f+3} = \frac{4-5f}{f+3}$$

$$m = \frac{4-5f}{f+3}$$

Functions

Instead of writing an equation as $y = \dots$ we can write it as

$$f(x) = \dots \text{ or } f : x \rightarrow \dots$$

This means that our INPUT is x and we SUBSTITUTE it in to the function f to find our OUTPUT y (or $f(x)$)

Example 1:

If $f(x) = 2x^2 + 3x$, find $f(-4)$.

$$\begin{aligned} f(-4) &= 2(-4)^2 + 3(-4) \\ &= 2(16) - 12 \\ &= 32 - 12 \\ &= 20 \end{aligned}$$

replace every x
in the function
with -4

Example 2:

If $f(x) = 2x + 3$, solve $f(x) = 6$.

$$\begin{aligned} 2x + 3 &= 6 \\ -3 &-3 \\ \hline 2x &= 3 \\ \frac{2x}{2} &= \frac{3}{2} \\ x &= 1.5 \end{aligned}$$

put the function
equal to 6 and then
solve the equation

Composite functions

If $f(x)$ and $g(x)$ are functions then $gf(x)$ is a COMPOSITE FUNCTION.

This means substitute $f(x)$ into the function g

Remember: always work out from the bracket!

1) Copy composite

2) Bring down inside

3) Bring down outside

4) Expand and simplify

$$fg(x)$$

$$f(\text{---}) =$$

NOTE: $g^2(x)$ is the
same as $gg(x)$

$$\text{If } f(x) = 3x - 2 \text{ and } g(x) = x^2 + 1$$

Example 1: Calculate: $fg(x)$

$$\begin{aligned} f(x^2 + 1) &= 3(x^2 + 1) - 2 \\ &= 3x^2 + 3 - 2 \\ &= 3x^2 + 1 \end{aligned}$$

sub $g(x)$ into $f(x)$

Example 2: Calculate: $gf(x)$

$$\begin{aligned} g(3x - 2) &= (3x - 2)^2 + 1 \\ &= 9x^2 - 6x - 6x + 4 + 1 \\ &= 9x^2 - 12x + 5 \end{aligned}$$

sub $f(x)$ into $g(x)$

Inverse functions

An inverse function reverses the effect of a function.

The inverse of $f(x)$ is written as $f^{-1}(x)$.

- 1) Write as $y =$
- 2) Swap x and y
- 3) Rearrange to make y the subject
- 4) $f^{-1}(x) =$

Example:

If $f(x) = \frac{5x - 6}{3}$ find $f^{-1}(x)$.

$$y = \frac{5x - 6}{3}$$

$$x = \frac{5y - 6}{3}$$

$$\times 3 \quad \times 3$$

$$3x = 5y - 6$$

$$+ 6 \quad + 6$$

$$\frac{3x + 6}{3} = \frac{5y}{3}$$

$$\frac{3x + 6}{3} = y$$

you need to recognise that
this means "find the inverse"

Answer: $f^{-1}(x) = \frac{3x + 6}{5}$

Iteration

Substituting in your previous answer to get an increasingly accurate estimate to the solution to an equation.

Example part a):

Show that the equation $x^3 + 2x = -5$

has a solution between $x = -2$ and $x = -1$.

$$f(x) = x^3 + 2x + 5$$

$$f(-2) = (-2)^3 + 2(-2) + 5 = -7$$

$$f(-1) = (-1)^3 + 2(-1) + 5 = 2$$

Change in sign solution between $x = -2$ and $x = -1$.

One is negative and one is positive therefore there is a change in sign

For part a)

1) Rearrange to $= 0$

2) Sub in x-values

3) "Change in sign solution between ..."

Example part c):

Starting with $x_0 = 4$, use the iteration formula $x_{n+1} = \sqrt{5x_n - 6}$ to find the values of x_1 , x_2 and x_3 .

$$x_0 = 4$$

$$x_1 = \sqrt{5(4) - 6} = 3.741657387$$

$$x_2 = \sqrt{5(\text{ans}) - 6} = 3.564868431$$

$$x_3 = \sqrt{5(\text{ans}) - 6} = 3.43865412$$

1) Sub in x_0

2) Sub answer using (Ans)

3) Repeat as necessary

Make sure you write out all the digits on the calculator display unless the question says otherwise!

NOTE: If asked to explain what the values of x_1 , x_2 and x_3 represent then write

"The equation was rearranged to give the iteration formula, and x_1 , x_2 and x_3 are increasingly accurate estimates of the solution"

Example 2: Finding a constant

A hot air balloon is descending.

The height of the balloon n minutes after it starts to descend is h_n metres.

The height of the balloon $(n+1)$ minutes after it starts to descend, h_{n+1} metres, is given by

$$h_{n+1} = K \times h_n + 20 \text{ where } K \text{ is a constant.}$$

The balloon starts to descend from a height of 1200 metres at 09 15

At 09 16 the height of the balloon is 1040 metres

Work out the height of the balloon at 09 18

solving for K , so At 09 16, the balloon is at 1040

$$h_{n+1} = K \times h_n + 20$$

$$(1040) = K \times (1200) + 20$$

$$\frac{1020}{1200} = \frac{1200K}{1200}$$

$$0.85 = K$$

$$h_{n+1} = 0.85 \times h_n + 20$$

h_n is now 1040

$$09\ 16\ h_0 = 1040$$

$$09\ 17\ h_1 = 0.85 \times (1040) + 20$$

$$h_1 = 904$$

$$09\ 18\ h_2 = 0.85 \times (\text{ANS}) + 20$$

$$= 788.4$$

Algebraic proof

Algebraic proof requires you to form, expand, simplify and factorise expressions

Algebraic expressions:

- An integer: n
- Two consecutive integers: $n, n+1, \dots$
- Even number: $2n$ Proof: "any multiple of 2 is always even"
- A different even number: $2m$
- Two consecutive even numbers: $2n, 2n+2, \dots$
- Odd number: $2n+1$ Proof: "any multiple of 2 is always even, so even + 1 = odd"
- A different odd number: $2m+1$
- Two consecutive odd numbers: $2n+1, 2n+3, \dots$

Example 1: Prove a multiple

Prove that $(3n+1)^2 - (3n-1)^2$ is a multiple of 6 for all positive integer values of n

$$9n^2 + 6n + 1 - (9n^2 - 6n + 1)$$

$$9n^2 + 6n + 1 - 9n^2 + 6n - 1$$

$$9n^2 + 6n - 9n^2 + 6n$$

$$12n = 6(2n)$$

$$\text{a multiple of 6}$$

(because we factorised by 6)

Steps

- 1) form your own expression?
- 2) expand
- 3) simplify
- 4) factorise
- 5) write conclusion

Example 2: Prove even

Prove that the difference between two different odd numbers is even

different odd numbers:

$$2n+1$$

$$2m+1$$

$$2n+1 - (2m+1)$$

$$2n+1 - 2m - 1$$

$$2n - 2m$$

$$2(n - m)$$

$$\text{any multiple of 2 is always even}$$

must use different letters!

Algebraic proof

Example 3: Prove odd

remember difference is big - small

Prove algebraically that the difference between the squares of any two consecutive numbers is always an odd number

consecutive numbers:

$$\begin{array}{|c|} \hline n \\ \hline n + 1 \\ \hline \end{array}$$

$$(n + 1)^2 - (n)^2$$

$$n^2 + 2n + 1 - n^2$$

$$2n + 1$$

any multiple of 2 is always even, so even + 1 = odd

Steps

- 1) form your own expression?
- 2) expand
- 3) simplify
- 4) factorise
- 5) write conclusion

Example 4: Prove square number

The product of two consecutive positive integers is added to the larger of the two integers. Prove that the result is always a square number.

consecutive positive integers:

$$\begin{array}{|c|} \hline n \\ \hline n + 1 \\ \hline \end{array}$$

product

add on larger expression

$$n(n + 1) + (n + 1)$$

$$n^2 + n + n + 1$$

$$n^2 + 2n + 1$$

$$(n + 1)(n + 1)$$

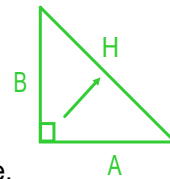
$$(n + 1)^2$$

square number

To prove the result is a square number, factorise into the same double bracket so that it can be written as ()²

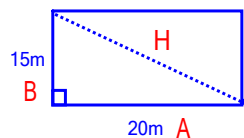
Pythagoras

- 1) Label triangle (Hypotenuse is opposite right angle)
- 2) Sub into formula: $H^2 = A^2 + B^2$
- 3) Rearrange and solve



Example:

A rectangular field is 20m long and 15m wide. Find the diagonal length of the field.



$$H^2 = 20^2 + 15^2$$

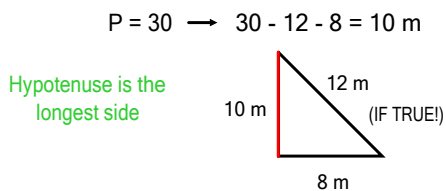
$$H^2 = 400 + 225$$

$$H^2 = 625$$

$$H = \sqrt{625} \quad H = 25\text{m}$$

Extension:

The perimeter of a triangle is 30m. It has two sides of lengths 8m and 12m. Is it a right-angled triangle? Justify your answer.



$$12^2 = 10^2 + 8^2$$

$$144 = 100 + 64$$

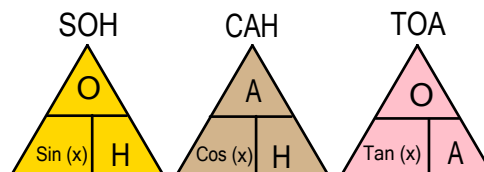
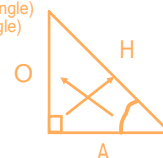
$$144 \neq 164$$

By Pythagoras' Theorem; it is not a right-angled triangle

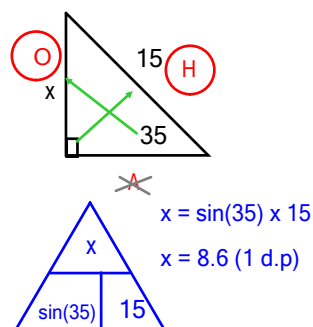
SOH CAH TOA

To be used with RIGHT ANGLED TRIANGLES only

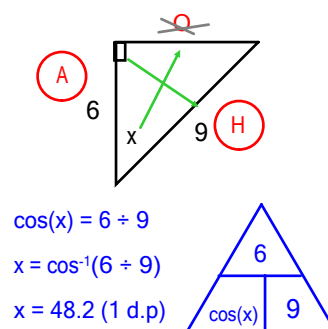
- 1) Label triangle (H, O, A)
H = hypotenuse (opposite right angle)
 O = opposite (opposite labelled angle)
 A = adjacent (next to labelled angle)
- 2) Circle what you have and what you want
- 3) Pick and draw ratio triangle
- 4) Form equation and solve



Example 1:
Find the missing length x:



Example 2:
Find the missing angle x:



Exact TRIG values

Trigonometry questions could appear on the non-calculator paper.

In this instance you will need to know the following exact values:

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm \infty$

Here is one way you can memorise the values for sin and cos.

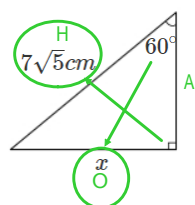
	0°	30°	45°	60°	90°
sin	0	1	2	3	4
cos	4	3	2	1	0

2

To find tan... you can use $\frac{\sin(\)}{\cos(\)}$

Example 1:

Find x without using a calculator



$$\sin(60) = \frac{\sqrt{3}}{2}$$



$$x = \frac{\sqrt{3}}{2} \times \frac{7\sqrt{5}}{1}$$

$$x = \frac{7\sqrt{3} \times \sqrt{5}}{2}$$

$$x = \frac{7\sqrt{15}}{2}$$

Example 2:

Find the exact value of $4\cos(30) + 3\tan(60)$ without using a calculator

$$\cos(30) = \frac{\sqrt{3}}{2}$$

$$4\left(\frac{\sqrt{3}}{2}\right) + 3(\sqrt{3})$$

$$\tan(60) = \sqrt{3}$$

$$2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$$

Sine rule

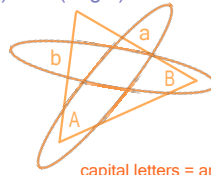
sine sausages!

- 1) Circle two pairs
- 2) Label thing you want to find a (side) or A (angle)

$$3) \text{ If finding angle: } \frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\text{If finding length: } \frac{a}{\sin A} = \frac{b}{\sin B}$$

- 4) Solve equation

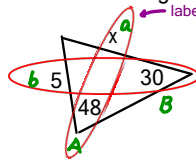


capital letters = angles

lower case = sides

Example 1:

Find the missing length in the shape below:



label missing length a

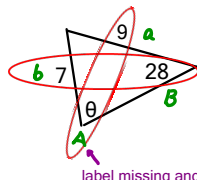
$$\frac{x}{\sin(48)} = \frac{5}{\sin(30)}$$

$$x = \frac{5}{\sin(30)} \times \sin(48)$$

$$x = 7.43 \text{ (2 d.p.)}$$

Example 1:

Find the missing angle in the shape below:



label missing angle A

$$\frac{\sin 9}{9} = \frac{\sin 28}{7}$$

$$\sin 9 = \frac{\sin 28}{7} \times 9$$

$$\theta = \sin^{-1}\left(\frac{\sin 28}{7} \times 9\right) = 37.1^\circ$$

HINT: if you can't straight away spot TWO PAIRS think about finding the third (missing) angle.

Cosine rule

cosine sausage!

- 1) Circle one pair
- 2) Label thing you want to find a (side) or A (angle) - this should be included in the pair

- 2) Sub values in cosine rule:

- To find missing angle

(three sides given)

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

- To find missing side

(enclosed angle)

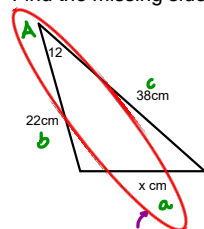
$$a^2 = b^2 + c^2 - 2bccos(A)$$

NOTE: b and c are the other lengths (order doesn't matter)

- 3) Solve

Example 1:

Find the missing side in the shape below:



label missing length a

$$a^2 = b^2 + c^2 - 2bccos(A)$$

$$x^2 = (22)^2 + (38)^2 - 2(22)(38)\cos(12)$$

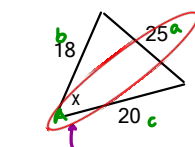
$$x^2 = 292.5372116$$

$$\sqrt{\quad} \quad \sqrt{\quad}$$

$$x = 17.1 \text{ (1 d.p.)}$$

Example 2:

Find the missing angle in the shape below:



label missing angle A

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos(x) = \frac{(18)^2 + (20)^2 - (25)^2}{2(18)(20)}$$

$$\cos x = \frac{11}{80}$$

$$x = \cos^{-1}\left(\frac{11}{80}\right) = 82.1^\circ \text{ (1 d.p.)}$$

Area of triangle

- For right-angled triangles:

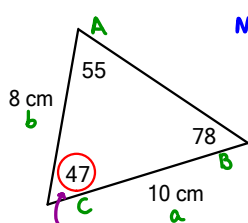
$$\frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

- For non-right-angled triangles:

$$\frac{1}{2} ab \sin(C) \quad \text{NOTE: } C \text{ is the enclosed angle}$$

Example1:

Find the area of the triangle:



this is the enclosed angle so label it C

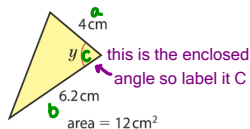
Non-right-angled so use $\frac{1}{2} ab \sin(C)$

$$= \frac{1}{2} (10)(8) \sin(47)$$

$$= 29.25 \text{ cm}^2$$

Example2:

Find the size of angle y:



this is the enclosed angle so label it C

$$\text{area} = \frac{1}{2} ab \sin C$$

$$12 = \frac{1}{2} (4)(6.2) \sin(y)$$

$$\frac{12}{12.4} = \frac{12.4}{12.4} \sin(y)$$

$$\frac{30}{31} = \sin(y)$$

$$y = \sin^{-1}\left(\frac{30}{31}\right) = 75.4^\circ \text{ (1 d.p.)}$$

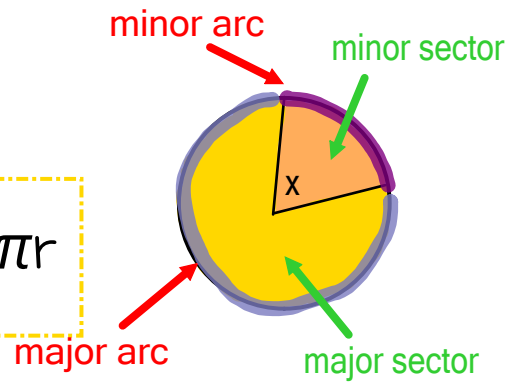
Arcs and sectors

Area of a sector:

$$\frac{\theta}{360^\circ} \times \pi r^2$$

Length of an arc:

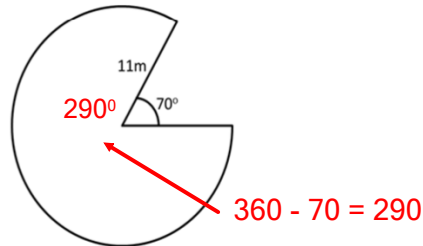
$$\frac{\theta}{360^\circ} \times 2\pi r$$



Example:

Find:

- the major arc length
- the major sector area

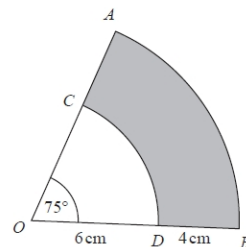


$$a) L = \frac{290}{360} \times 2\pi \times 11 = 55.7 \text{ m}$$

$$b) A = \frac{290}{360} \times \pi \times 11^2 = 306.2 \text{ m}^2$$

Shaded areas

- Identify your two shapes (large and small)
- Find the area of each
- Subtract areas



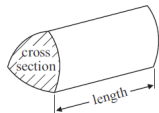
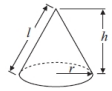
Example:

Find the shaded area. Give your answer to 1 d.p.

$$\begin{aligned} & \text{Area of large sector} - \text{Area of small sector} = 41.9 \text{ cm}^2 \\ & \frac{75}{360} \times \pi \times 10^2 = \frac{125\pi}{6} \quad \frac{75}{360} \times \pi \times 6^2 = \frac{15\pi}{2} \\ & \frac{125\pi}{6} - \frac{15\pi}{2} = 41.9 \text{ cm}^2 \end{aligned}$$

NOTE: you could also be asked to find shaded area with a combination of sectors, circles, triangles and squares. Use the steps above and draw diagrams to help you.

3D shapes: surface area & volume

Volume of sphere = $\frac{4}{3}\pi r^3$ Surface area of sphere = $4\pi r^2$ 	Volume of prism = area of cross section $\times$ length 	Volume of cone = $\frac{1}{3}\pi r^2 h$ Curved surface area of cone = $\pi r l$ 
---	---	---

Example: NOTE: when estimating π (3.14159....) use 3

Jan has some metal that she is going to make into solid metal spheres.

Each sphere will have a radius of 2.15 cm.

Jan has 1490 cm² of metal.

Work out an estimate for the number of spheres that Jan can make.

Surface area of sphere =
 $4\pi r^2$
 $4(3)(2^2)$
 $48 \text{ cm}^2 \Rightarrow$ round again **50**

round to 1 s.f.
 $2.15 \Rightarrow 2$
 $1490 \Rightarrow 1000$
 $\pi \Rightarrow 3$

If you are filling something or splitting it divide

$$1000 \div 50 = 20$$

ANSWER: Jan could make 20 spheres.

Rates of change: volume

- 1) Find volume of shape to be filled
- 2) Convert from m^3 to litres ($1\text{m}^3 = 1000$ litres)
- 3) Draw ratio table to compare time with litres

Example:

The diagram shows a swimming pool in the shape of a prism.

The swimming pool is empty.

The swimming pool is filled with water at a **constant rate of 50 litres per minute**,

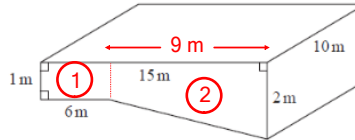
Work out how long it will take for the swimming pool to be completely full of water. Give your answer in hours.

Area of cross section:

1) $1 \times 6 = 6 \text{ m}^2$

2) $\frac{1}{2} \times 9 \times (1 + 2) = 13.5 \text{ m}^2$

19.5 m^2



Volume of prism: $19.5 \times 10 = 195 \text{ m}^3$

Convert to litres: $195 \times 1000 = 195\,000$ litres

	mins	litres	
$\div 50$	1	50	$\div 50$
	0.02	1	
$\times 195000$	3900	195 000	$\times 195000$

Change to hours: $3900 \div 60 = 65$ hours

Similar triangles

Two triangles are similar if all corresponding angles are equal and corresponding sides are in the same proportion.

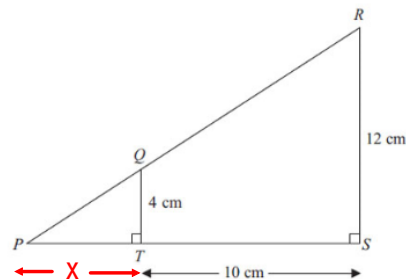
HINT: look out for "triangles inside triangles"

Example:

PQR and PTS are straight lines.

Angle PTQ = Angle PSR = 90°

Work out the length of PT.



Divide

	small	scale factor	big
L	4	$r = \frac{12}{4} = 3$	12
L	x	$r = 3$	$10 + x$

Multiply

$x \times 3 = 10 + x$

$3x = 10 + x$

$-x \quad -x$

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

$\therefore PT = 5$

Similar shapes

Whenever a question involves SIMILAR SHAPES use a L.A.V
(length, area, volume) table

NOTE: you may sometimes need L.L.A.V tables if more than
one length is being compared.

Example:

Two solid cones are mathematically similar.

Cone A has a volume of 120cm^3 .

Cone B has a volume of 960cm^3 .

The surface area of Cone B is 108cm^2 .

Work out the surface area of Cone A.

HINT: find s.f. ratios as fractions
rather than decimals where
possible.

	A	scale factor	B	
L		$r = \sqrt[3]{8} = 2$		
A	27	$r^2 = 2^2 = 4$	108	Answer: 27cm^2
V	120	$r^3 = \frac{960}{120} = 8$	960	

Divide

Multiply

Combinations

If the order is important - don't divide

If the order is NOT important - divide by 2

Example:

There are 17 men and 26 women in a choir

The choir are singing in a concert.

a) One of the men and one of the women are going to be chosen to make a pair to sing the first song. Work out the number of different pairs that can be chosen.

$$\begin{array}{cc} \text{M} & \text{W} \\ 17 \times 26 & = 442 \end{array}$$

Order is important as first has to be man so we don't divide

b) Two of the men are going to be chosen to make a pair to sing the second song. Work out the number of different pairs that can be chosen.

$$\begin{array}{cc} \text{M} & \text{M} \\ 17 \times 16 & = 272 \end{array}$$

Order is NOT important as it doesn't say which man has to go first so we divide

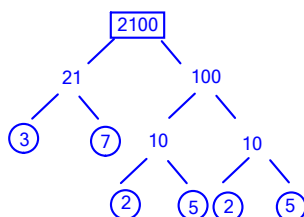
$$272 \div 2 = 136$$

Product of prime factors, HCF, LCM

Prime numbers: 2, 3, 5, 7, 11, 13, 17, ...

Example 1:

Express 2100 as a product of its prime factors in index notation.



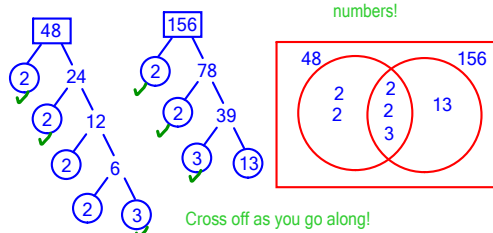
$$2 \times 2 \times 3 \times 5 \times 5 \times 7$$

$$2^2 \times 3 \times 5^2 \times 7 \text{ (index notation)}$$

Example 2:

Find the HCF and LCM of 48 and 156

Use a Venn diagram for big numbers or more than 2 numbers!



Cross off as you go along!



$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 13 = 624$$



$$\text{HCF} = 2 \times 2 \times 3 = 12$$

Standard form

$$A \times 10^n$$

where A is between 1 and 10 and n tells you how many places to move the decimal point. NOTE: if n is positive this represents a very big number, if n is negative this represents a very small number.

Example:

a) Write 4.2×10^4 as an ordinary number.

4 . 2 0 0 0 .

Decimal point moves right to make a BIG number

Answer: 42000

b) Write the number 0.000354 in standard form.

0 . 0 0 0 3 . 5 4

n is - 4 as number was SMALL

Answer: 3.54×10^{-4}

- Multiplication/Division with Standard Form:

multiply/divide numbers and add/subtract powers of 10

- Addition/Subtraction with Standard Form:

convert values to ordinary numbers first

Standard form - calculations

Addition & Subtraction:

- 1) Convert to ordinary numbers
- 2) add/subtract
- 3) convert back to standard form if question asks

Example:

Calculate $4.2 \times 10^4 + 3.75 \times 10^3$ and leave your answer in standard form.

4.2000.

3.750.

42000

3750

45750

Answer: 4.575×10^4



Multiplication & Division (shortcut using index laws):

- 1) $\times/\div$ normal numbers
- 2) $+/-$ powers of 10
- 3) check answer is still in standard form

Example:

Calculate $(4 \times 10^4) \times (6 \times 10^3)$ and leave your answer in standard form.

$$4 \times 6 = 24$$

$$10^4 \times 10^3 = 10^{4+3} = 10^7$$

$$= 24 \times 10^7$$

← not in standard form as 1st number isn't between 1 and 10

$$= 2.4 \times 10^8$$

Surds: simplifying

- 1) Look for the biggest square factor
- 2) Write square factor first
- 3) Square root

Example 1:

Simplify $\sqrt{20}$

$$\sqrt{4} \times \sqrt{5}$$

$$2 \times \sqrt{5}$$

$$\boxed{2\sqrt{5}}$$

Example 2:

Simplify $\sqrt{32}$

$$\sqrt{16} \times \sqrt{2}$$

$$4 \times \sqrt{2}$$

$$\boxed{4\sqrt{2}}$$

$\sqrt{16}$ is the biggest square factor!

Example 3

Simplify $\sqrt{72}$

$$5 \times \sqrt{36} \times \sqrt{2}$$

$$5 \times 6 \times \sqrt{2}$$

$$\boxed{30\sqrt{2}}$$

Square Numbers:

1

4

9

16

25

36

49

64

Surds: CalculationsMultiplication & Division

- 1) $\times/\div$ normal numbers and surds separately
- 2) simplify if possible

Example 1:

Simplify $4/\sqrt{10} \times 3/\sqrt{5}$

$$\begin{aligned}
 &4 \times 3 \times \sqrt{10} \times \sqrt{5} \\
 &12 \times \sqrt{50} \\
 &12 \times \sqrt{25} \times \sqrt{2} \\
 &12 \times 5 \times \sqrt{2} \\
 &\boxed{60\sqrt{2}}
 \end{aligned}$$

Addition & Subtraction

- 1) simplify surds to get same number under $\sqrt{\quad}$
- 2) add/subtract like surds

Example 1:

Simplify $2/\sqrt{3} + 5/\sqrt{3}$ $\leftarrow$ same number under $\sqrt{\quad}$ so can be added together

$$= 7\sqrt{3}$$

Example 2

Simplify $5/\sqrt{8} + 7/\sqrt{18}$ $\leftarrow$ simplify both surds to get same number under $\sqrt{\quad}$

$$\begin{array}{ll}
 5\sqrt{8} & 7\sqrt{18} \\
 5 \times \sqrt{4} \times \sqrt{2} & 7 \times \sqrt{9} \times \sqrt{2} \\
 5 \times 2 \times \sqrt{2} & 7 \times 3 \times \sqrt{2} \\
 = 10\sqrt{2} & = 21\sqrt{2} \\
 \\
 10\sqrt{2} + 21\sqrt{2} & \\
 \boxed{= 31\sqrt{2}} &
 \end{array}$$

Square Numbers:

1
4
9
16
25
36
49
64

Surds: expanding

$$\sqrt{a} \times \sqrt{a} = a$$

Example 1:

$$\sqrt{5}(3 + \sqrt{5})$$

$$= 3\sqrt{5} + 5$$

Example 2

$$(3 + \sqrt{2})(3 - \sqrt{2})$$

$$\begin{array}{cc}
 & 3 & +\sqrt{2} \\
 3 & \boxed{9} & +3\sqrt{2} \\
 -\sqrt{2} & -3\sqrt{2} & -2
 \end{array}$$

$$9 + \cancel{3\sqrt{2}} - \cancel{3\sqrt{2}} - 2$$

$$\boxed{= 7}$$

Square Numbers:

1
4
9
16
25
36
49
64

Surds: Expanding and simplifying

Always look for surds you can simplify before starting a question!

Example 1:

Show that $(5 - \sqrt{8})(7 + \sqrt{2})$ can be written in the form $a + b\sqrt{2}$

$\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

5	$-2\sqrt{2}$
$7 \cdot 35$	$-14\sqrt{2}$
$+ \sqrt{2} \cdot 5\sqrt{2}$	$-2(-4)$

$35 - 14\sqrt{2} + 5\sqrt{2} - 4$
 $31 - 9\sqrt{2}$

Answer: $a = 31$
 $b = -9$

Example 2:

Given that $(5 - \sqrt{x})^2 = y - 20\sqrt{2}$ where x and y are positive integers, find the value of x and the value of y .

y is the number family on its own
 $-20\sqrt{2}$ is the surd family

5	$-\sqrt{x}$
$5 \cdot 25$	$-5\sqrt{x}$
$-\sqrt{x} \cdot -5\sqrt{x}$	$+x$

$25 - 10\sqrt{x} + x$

$$25 - 10\sqrt{x} + x = y - 20\sqrt{2}$$

$-10\sqrt{x} = -20\sqrt{2}$
 $\frac{-10\sqrt{x}}{-10} = \frac{-20\sqrt{2}}{-10}$
 $\sqrt{x} = 2\sqrt{2}$
 $x = 4(2)$
 $x = 8$

$25 + x = y$
 $25 + 8 = y$
 $33 = y$

Surds: rationalising the denominator

"Rationalise" means get rid of the surd from the denominator

Remember $\sqrt{a} \times \sqrt{a} = a$

Example (1 term in denominator):

Write $\frac{8}{\sqrt{2}}$ in its simplest form.

Multiply top and bottom of fraction by surd

$$\frac{8}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

Example (2 terms in denominator):

Write $\frac{12}{3 + \sqrt{3}}$ in its simplest form.

Multiply top and bottom of fraction by the conjugate:

If $(a + \sqrt{b})$ $(a - \sqrt{b})$

If $(a - \sqrt{b})$ $(a + \sqrt{b})$

$\frac{12}{3 + \sqrt{3}} \times \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$

swap the sign in the denominator to get the conjugate

Numerator	Denominator						
$12(3 - \sqrt{3})$	$3 + \sqrt{3}$						
$36 - 12\sqrt{3}$	<table border="1"> <tr> <td>3</td> <td>9</td> <td>$+3\sqrt{3}$</td> </tr> <tr> <td>$-\sqrt{3}$</td> <td>$-3\sqrt{3}$</td> <td>-3</td> </tr> </table>	3	9	$+3\sqrt{3}$	$-\sqrt{3}$	$-3\sqrt{3}$	-3
3	9	$+3\sqrt{3}$					
$-\sqrt{3}$	$-3\sqrt{3}$	-3					
	$9 - 3 = 6$						

$$= \frac{36 - 12\sqrt{3}}{6}$$

$$= \frac{6(6 - 2\sqrt{3})}{6}$$

$$= 6 - 2\sqrt{3}$$

Recurring decimals

- 1) Let x = recurring decimal
- 2) Find $10x$, $100x$, $1000x$ etc.
- 3) Look after decimal point to spot a matching pair
- 4) Subtract the equations, divide and simplify

Example:

Express $0.\dot{3}\dot{2}\dot{5}$ as a fraction in its simplest form.

$$x = 0.3252525 \dots$$

$$10x = 3.\underline{252525} \dots *$$

$$100x = 32.52525 \dots$$

$$1000x = 325.\underline{25252} \dots *$$

$$1000x = 325.\underline{2525} \dots$$

$$\begin{array}{r} - \quad 10x = 3.\underline{2525} \dots \\ \hline 990x = 323 \end{array}$$

$$\text{ANSWER: } x = 323/990$$

Percentage increase/decrease

$$\text{Original Amount} \times \left(\frac{100 \pm \%}{100} \right)^{\text{Time}} = \text{New Amount}$$

The bit in brackets is the multiplier:

- Add for % increase

- Subtract for % decrease

Example 1 (calculator):

Increase £55 by 3.2%

original

$$55 \times \left(\frac{100 + 3.2}{100} \right)$$

$$= 55 \times 1.032$$

$$= \pounds 56.76$$

Example 2 (non-calculator):

Decrease £64 by 16%

For non-calc, find 10%, 5%, 1% and combine to find the % you need, then add or subtract it from the total.

$$10\% = 6.40$$

$$5\% = 3.20$$

$$1\% = 0.64$$

$$\hline 16\% = 10.24$$

$$\begin{array}{r} 64.00 \\ - 10.24 \\ \hline \end{array}$$

$$\pounds 53.76$$

Compound percentages

$$\text{Original Amount} \times \left(\frac{100 \pm \%}{100} \right)^{\text{Time}} = \text{New Amount}$$

Example 1

The value of a machine when new is £8000. The value of the machine depreciates by 10% each year. What is its value after 3 years?

$$\begin{aligned} & 8000 \times \left(\frac{100-10}{100} \right)^3 \\ & = 8000 \times 0.9^3 \\ & = \pounds 5832 \end{aligned}$$

Example 2 (more than one interest rate):

Sally invests £510 in a bank account with a compound interest rate of 2% for 2 years. After 2 years the interest rate increases to 5%. How much money will she have after 5 years?

$$\begin{aligned} & 510 \times \left(\frac{100+2}{100} \right)^2 \times \left(\frac{100+5}{100} \right)^3 \\ & = \pounds 614.24 \end{aligned}$$

Example 3 (finding the number of years): Use trial and improvement

Tariq deposits £3000 in a bank account with a compound interest rate of 4% per annum. He wants to leave the money in the account until there is at least £3500. How many years does he need to leave the money in the account?

$$\begin{aligned} \text{trial 1: } & 3000 \times \left(\frac{100+4}{100} \right)^1 = \pounds 3120 \\ \text{trial 2: } & 3000 \times \left(\frac{100+4}{100} \right)^2 = \pounds 3244.80 \\ \text{trial 3: } & 3000 \times \left(\frac{100+4}{100} \right)^3 = \pounds 3374.59 \\ \text{trial 4: } & 3000 \times \left(\frac{100+4}{100} \right)^4 = \pounds 3509.58 \quad \checkmark \\ & \text{Answer: 4 years} \end{aligned}$$

Calculating percentage change

$$\text{Original Amount} \times \left(\frac{100 \pm \%}{100} \right)^{\text{Time}} = \text{New Amount}$$

Example 1

In a storm 144 fruit trees were left standing out of 180 fruit trees in an orchard. What is the percentage decrease in the number of trees?

$$\begin{aligned} & \frac{144}{180} \times \left(\frac{100-x}{100} \right) = \frac{144}{180} \\ & \frac{100-x}{100} = 0.8 \\ & \times 100 \quad \times 100 \\ & 100-x = 80 \\ & +x \quad +x \\ & 100 = 80+x \\ & -80 \quad -80 \\ & 20 = x \\ & \text{Answer: } 20\% \text{ decrease} \end{aligned}$$

Example 2 (more than one interest rate):

Toby invests £6000 for 4 years. the investment gets compound interest of x% per annum. At the end of 4 years the investment is worth £6885.12. Work out the value of x to 1 decimal place.

$$\begin{aligned} & \frac{6000}{6000} \times \left(\frac{100+x}{100} \right)^4 = \frac{6885.12}{6000} \\ & \left(\frac{100+x}{100} \right)^4 = 1.14752 \\ & \sqrt[4]{\quad} \quad \sqrt[4]{\quad} \\ & \frac{100+x}{100} = 1.034999... \\ & \times 100 \quad \times 100 \\ & 100+x = 103.4999 \\ & -100 \quad -100 \\ & x = 3.4999 \\ & \text{Answer: } 3.5\% \end{aligned}$$

Reverse percentages

$$\text{Original Amount} \times \left(\frac{100 \pm \%}{100} \right)^{\text{Time}} = \text{New Amount}$$

Example 1: ↑ decrease = -

A car depreciates in value by 30% during its first year. Its value now is £8960. What was its original price?

↓
new

← call this x

$$x \times \left(\frac{100 - 30}{100} \right) = 8960$$

$$x \times \frac{0.7}{0.7} = \frac{8960}{0.7}$$

$$x = 12800$$

answer: £12800

Example 2 (reverse compound):

A newspaper increased its circulation by 3% each year for 2 years and the new number sold was 58195. What was it before the increase?

↙ new

↘ x

$$x \times \left(\frac{100 + 3}{100} \right)^2 = 58195$$

$$x \times \frac{1.03^2}{1.03^2} = \frac{58195}{1.03^2}$$

$$x = 54,854$$

Speed, distance, time

1) Draw table

2) Use triangle to find missing values



Example 1:

Harry travels from Appleton to Brockley at an average speed of 50 mph.

He then travels from Brockley to Cantham at an average speed of 70 mph.

Harry takes a total time of 5 hours to travel from Appleton to Cantham. The distance from Brockley to Cantham is 210 miles.

Calculate Harry's average speed for the total distance travelled from Appleton to Cantham.

	S	D	T
A - B	50	$50 \times 2 = 100$	$5 - 3 = 2$
B - C	70	210	$210 \div 70 = 3$
Total	$310 \div 5 = 62$	$100 + 210 = 310$	5

Harry drove at an average speed of 62 mph over the whole journey.

Example 2:

Edith leaves her home at 10:40 am.

She travels 60 miles at an average speed of 25 mph.

At what time does she finish her journey?

NOTE: make sure you use the correct units!



S = 25 mph D = 60 miles T = ?

$T = 60 \div 25 = 2.4$ hours

Change to minutes: $2.4 \times 60 = 144$ mins
= 2 hrs 24 mins

On a calculator paper hit the comma button at this point!



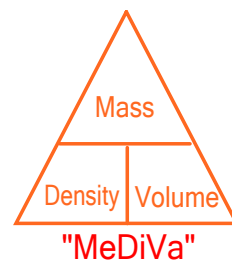
Edith finishes at 13:04

Never add or subtract the speed column

Mass, density, volume

1) Draw table

2) Use triangle to find missing values



Example:

Hannah mixes 150g of metal A and 150g of metal B to make 300g of an alloy. ——— total

Metal A has a density of 19.3 g/cm^3 .

Metal B has a density of 8.9 g/cm^3 .

Work out the density of the alloy.

Never add or subtract the density column

	D	M	V
Metal A	19.3	150	$150 \div 19.3 = 7.77202...$
Metal B	8.9	150	$150 \div 8.9 = 16.85393...$
Total	$300 \div 24.62595... = 12.182269503$	300	$7.77... + 16.85 = 24.62595...$

Answer: 12.18 g/cm^3

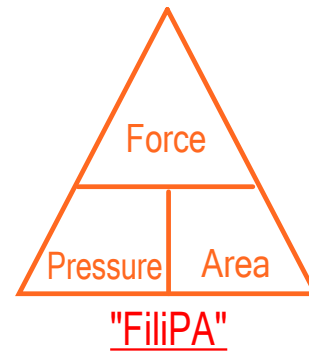
Force, pressure, area

Example:

A box exerts a force of 140 newtons on a table.

The pressure on the table is 35 newtons/m².

Calculate the area of the box that is in contact with the table.



$$F = 140 \quad P = 35 \quad A = ?$$

$$A = F \div P$$

$$= 140 \div 35$$

$$= 4\text{m}^2$$

AVERAGES: 'Mean' Mean

Example:

This year two classes, A and B, will be taking the

November exam. *number*

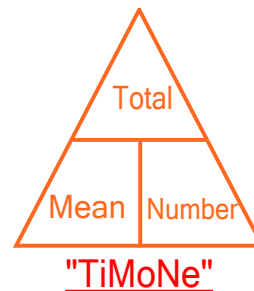
There are 20 students in class A.

There are 47 students in total taking the exam.

On the recent mock paper the mean mark for class A was 72.

The mean mark across both classes was 68.

Find the mean mark for class B.



	Mean	Number (in class)	Total
Class A	72	20	$72 \times 20 = 1440$
Class B	$1736 \div 27 = 65$	$47 - 20 = 27$	$3196 - 1440 = 1756$
Total	68	47	$68 \times 47 = 3196$

NEVER add or subtract in
the mean column

Answer = 65

Velocity-Time graphs: distance

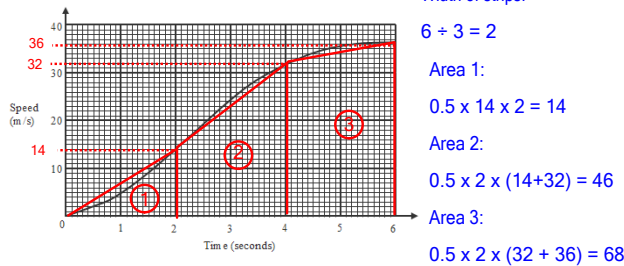
The AREA UNDER a velocity-time graph represents the DISTANCE TRAVELLED.

- 1) Calculate width of strip = time ÷ no. of strips
- 2) Add strips of equal distance to your graph and connect the top of the strips with a straight line
- 3) Find the area of each region **NOTE: area of trapezium = $\frac{1}{2}(a + b)h$**
- 4) Add the areas together to find an estimate for total distance travelled

Example:

The graph shows how a car's speed in m/s varies in the first 6 seconds after the car moves from rest.

Use 3 equal strips on the graph to estimate the distance travelled in the first 6 seconds.



If asked whether your estimate is an overestimate or underestimate ...

- UNDERESTIMATE: because area between curve and trapeziums is not included
- OVERESTIMATE: because ~~area~~ area between curve and trapezium is included

Velocity-Time graphs: acceleration

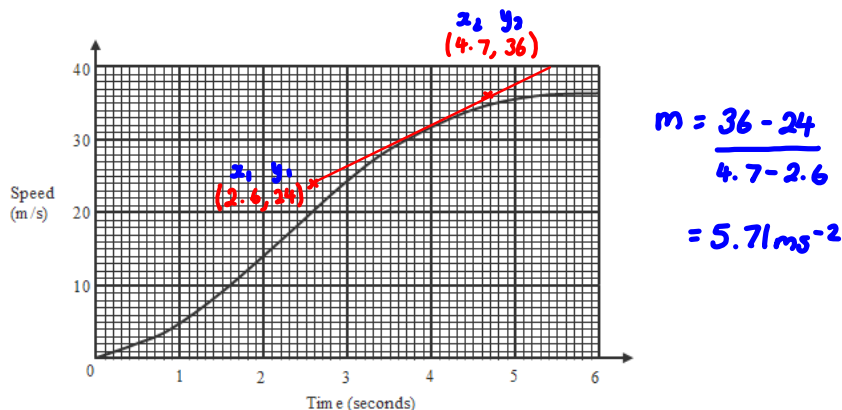
The GRADIENT of a velocity-time graph represents the ACCELERATION

- 1) Draw tangent at the specified time
- 2) Pick two clear points on the tangent (one on either side of original point)
- 3) Calculate gradient using $\frac{y_2 - y_1}{x_2 - x_1}$

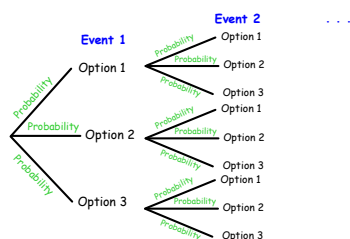
Example:

The graph shows how a car's speed varies in m/s in the first 6 seconds after the car moves from rest.

Estimate the acceleration of the car at $t = 4$ seconds.



Probability Trees



Option 1 AND Option 2
Multiply probabilities

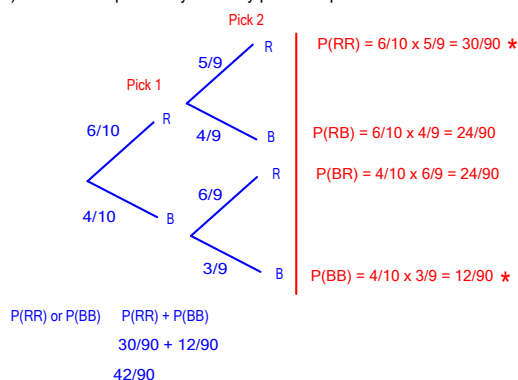
Option 1 OR Option 2
Add probabilities

At least x x or more
At most x up to x

Example:

There are 10 pens in a box. 6 are red and the rest are blue. Amy picks two pens from the box. **NOTE: the pen has NOT been replaced!**

- Draw a tree diagram to represent this information
- Work out the probability that Amy picks two pens of the same colour



Probability Trees: Algebraic

Sometimes it can be quicker NOT to draw the whole tree diagram but rather consider key options.

Example 1:

There are some red counters and some white counters in a bag.

At the start 7 of the counters are red and the rest are white.

Alfie takes at random a counter from the bag, he does not replace it.

Alfie takes at random another counter from the bag.

The probability that the first counter Alfie takes is white and the second counter Alfie takes is red is $\frac{21}{80}$.

Work out the number of white counters in the bag at the start.

red = 7, bag = x, white = x - 7

first counter not replaced!

$$\frac{x-7}{x} \times \frac{7}{x-1} = \frac{21}{80}$$

$$\frac{x-7}{x} \times \frac{7}{x-1} = \frac{21}{80}$$

$$\frac{7(x-7)}{x(x-1)} = \frac{21}{80}$$

$$\frac{7x-49}{x^2-x} = \frac{21}{80}$$

$$80(7x-49) = 21(x^2-x)$$

$$560x - 3920 = 21x^2 - 21x$$

$$-560x + 3920 = 21x^2 - 581x$$

$$0 = 21x^2 - 581x + 3920$$

$$3x^2 - 83x + 560 = 0$$

using quadratic formula!

$$x = \frac{-(-83) \pm \sqrt{(-83)^2 - 4(3)(560)}}{2(3)}$$

$$x = 16$$

bag = 16

$$\text{white} = x - 7 = 16 - 7 = 9$$

∴ There were 9 red counters

x = 11.666... must have a whole number of counters!

Probability Trees Algebraic

Example 2: with RATIO

John has an empty box.

He puts some red counters and some blue counters into the box.

The ratio of the number of red counters to the number of blue counters is 1:4

Linda takes at random two counters from the box

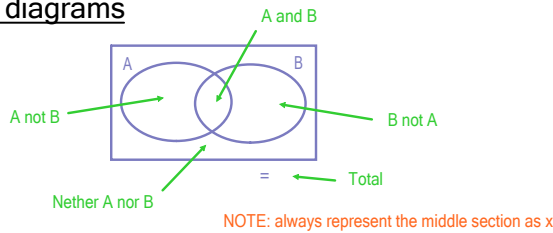
The probability that she takes 2 red counters is $\frac{6}{155}$.

How many counters did John put into the box? *not replaced!*

$$\begin{aligned}
 &R:B \\
 &1x:4x \rightarrow \text{total} = 5x \\
 &\frac{x}{5x} \times \frac{x-1}{5x-1} = \frac{6}{155} \\
 &\frac{x(x-1)}{5x(5x-1)} = \frac{6}{155} \\
 &\frac{x-1}{25x-5} = \frac{6}{155} \\
 &155(x-1) = 6(25x-5) \\
 &155x - 155 = 150x - 30 \\
 &-150x \quad -150x \\
 &5x - 155 = -30 \\
 &\quad +155 \quad +155 \\
 &\frac{5x}{5} = \frac{125}{5} \\
 &x = 25
 \end{aligned}$$

John put a total of $5x$ counters in the box,
 so $5x = 5(25) = \underline{\underline{125 \text{ counters}}}$

Venn diagrams



Example:

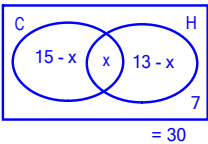
A class of 30 students are asked whether they watched Casualty or Holby City last week.

15 students watched Casualty.

13 students watched Holby city.

7 students watched neither programme.

How many students watched both?



Everything on the diagram adds up to 30

$$15 - x + x + 13 - x + 7 = 30$$
$$35 - x = 30$$
$$x = 5$$

$\{A \cup B\}$	A or B or both	Everything shaded	
$\{A \cap B\}$	A and B	Double shaded	
$\{A^c\}$	not A	Everything not shaded	
$\{A B\}$	A given B	$\frac{\text{Double Shaded}}{\text{Everything Shaded}}$	

Error Intervals: Truncating

Truncating a number is like cutting a number without rounding

Example 1: Truncate 3.7892 to 2 decimal places.

Answer: 3.78 (don't round!)

Example 2:

Lola truncates a number, x , to 1 d.p.

The result is 6.2

Write down the error interval for x .

Steps

1) Make a list of possible numbers before you decide on an error interval

2) Write in the form $\dots \leq x < \dots$

Imagine Lola has looked at her calculator and then truncated this number, what digits might she have seen in the decimal places?

6.200000111.....

6.249863896.....

6.255555555.....

6.299998989....

Answer: $6.2 \leq x < 6.3$

Error intervals: Rounding

1) Find the upper and lower bound

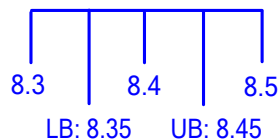
2) Write in the form $\dots \leq x < \dots$

Example 1:

Tim rounds a number, x , to 1 d.p.

The result is 8.4

Write down the error interval for x .



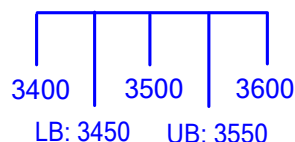
Answer: $8.35 \leq x < 8.45$

Example 2:

Louise rounds a number, m , to 2 s.f.

The result is 3500

Write down the error interval for m .



Answer: $3450 \leq m < 3550$

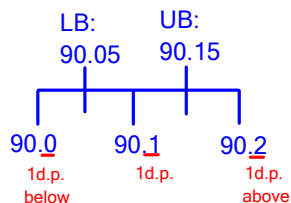
Bounds: Finding UB and LB

If you see the words "correct to the nearest ..."
that means its a BOUNDS question!

Example 1:

A value is rounded to 90.1 correct to the nearest 1 d.p.

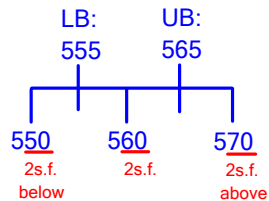
Find the lower bound and upper bound.



Example 2

A value is rounded to 560 correct to the nearest 2 s.f.

Find the lower bound and upper bound.



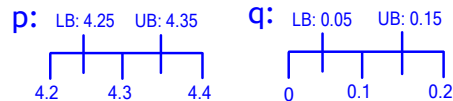
Bounds: calculating

	S		A		M		D		R		I		B
	+ / -		x / ÷		x / ÷		x / ÷		x / ÷		x / ÷		()
overall UB	UB + UB		UB - LB		UB x UB		$\frac{UB}{LB}$		UB ²		$\sqrt{UB}$		(lost)
overall LB	LB + LB		LB - UB		LB x LB		$\frac{LB}{UB}$		LB ²		$\sqrt{LB}$		(lost)

Example 1:

The value of p is 4.3cm and the value of q is 0.4cm

Both p and q are given correct to the nearest 0.1cm

Work out the upper bound for r when $r = p + \frac{1}{q}$ 

$$r = p + \frac{1}{q}$$

for addition: UB = LB + UB for division: UB = $\frac{UB}{LB}$

$$UB_r = UB_p + \frac{1}{LB_q}$$

$$UB_r = 4.35 + \frac{1}{0.05}$$

$$= 24.35$$

Bounds: suitable degree of accuracy

- 1) bounds
- 2) formula
- 3) sub in
- 4) calculate UB and LB
- 5) "UB and LB round to ___ to ___"

Example 1:

$$m = \frac{\sqrt{s}}{t}$$

s = 3.47 correct to 2 decimal places

t = 8.132 correct to 3 decimal places

By considering bounds, work out the value of m to a suitable degree of accuracy.

You must show all your working and give a reason for your final answer.

S: $\begin{array}{ccc} 3.46 & 3.47 & 3.48 \\ | & | & | \\ 3.465 & 3.475 & \end{array}$

t: $\begin{array}{ccc} 8.131 & 8.132 & 8.133 \\ | & | & | \\ 8.1315 & 8.1325 & \end{array}$

$M_{UB} = \frac{UB}{LB} = \frac{\sqrt{3.475}}{8.1315} = \frac{0.22912486243}{0.2291}$

$M_{LB} = \frac{LB}{UB} = \frac{\sqrt{3.465}}{8.1325} = \frac{0.2288903839}{0.2291}$

Conclusion:
Both UB and LB round to 0.229 to 3 decimal places

Copy all digits
look for the highest degree of accuracy that both numbers round to

Averages: estimating the mean

THE SUM OF THE VALUES DIVIDED BY THE FREQUENCY

Ungrouped data : Total $fx \div$ total f Grouped data: Total $fm \div$ total f
(where m is the midpoint)

Example:

The table shows the heights of 50 students.

Height, h , cm	Frequency	midpoint (m)	freq x midpoint (fm)
$110 \leq h < 120$	3	$(110+120) \div 2 = 115$	$3 \times 115 = 345$
$120 \leq h < 130$	8	125	$8 \times 125 = 1000$
$130 \leq h < 140$	9	135	$9 \times 135 = 1215$
$140 \leq h < 150$	23	145	$23 \times 145 = 3335$
$150 \leq h < 160$	7	155	$7 \times 155 = 1085$

$$\text{Total } fm = 6980$$

$$\text{Total } f = 50$$

$$\text{Mean: } 6980 \div 50 = 139.6 \text{ (to 1 d.p.)}$$

NOTE: Calculating the mean from a grouped frequency table is always an estimate because we have used the midpoint as we don't have the exact values for the data

Averages: estimating the median

NOTE: this kind of question may be asked in conjunction with Histograms.

- 1) Add cumulative frequency column
- 2) Find median value ($n \div 2$)
- 3) Find median class (where does the middle value lie?)
- 4) Draw diagram with frequency above and class interval below
- 5) $LB + \left(\frac{\text{distance into group}}{\text{frequency of group}} \times \text{class width} \right)$

Example:

The table shows some information about the time, in minutes, taken by a group of students to travel to college in one week.

Find an estimate for the median time.

Time (m minutes)	Frequency	c.f
$0 \leq m < 20$	20	20
$20 \leq m < 30$	30	50
$30 \leq m < 40$	45	95
$40 \leq m < 60$	60	155
$60 \leq m < 100$	48	203

median: $203 \div 2 = 101.5^{\text{th}}$ value which lies in the interval $40 \leq m < 60$

distance into group = $101.5 - 95 = 6.5$

$LB + \left(\frac{\text{distance into group}}{\text{frequency of group}} \times \text{class width} \right)$

$40 + \left(\frac{6.5}{60} \times 20 \right) = 42.2 \text{ (1 dp)}$

NOTE: you can use this formula for the lower quartile and the upper quartile, you will just need to use $n \div 4$ for the LQ and $(n \div 4) \times 3$ for the UQ

Frequency polygon

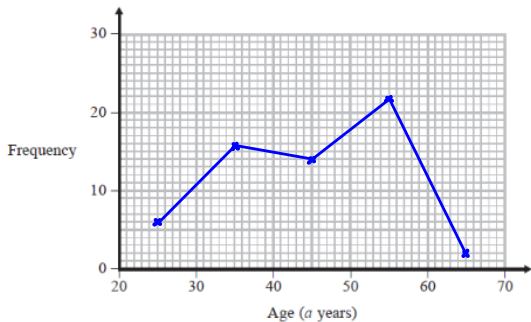
- 1) Find the midpoints
- 2) Plot coordinates using frequency and midpoint
- 3) Join points, using a ruler, starting from first coordinate and finishing at last (do NOT draw a line connecting first and last point!)

Example:

The table shows information about the ages of 60 teachers.

Draw a frequency polygon to represent this information.

Age (<i>a</i> years)	Frequency	Mid-point
$20 < a \leq 30$	6	$(20+30) \div 2 = 25$
$30 < a \leq 40$	16	$(30+40) \div 2 = 35$
$40 < a \leq 50$	14	$(40+50) \div 2 = 45$
$50 < a \leq 60$	22	$(50+60) \div 2 = 55$
$60 < a \leq 70$	2	$(60+70) \div 2 = 65$



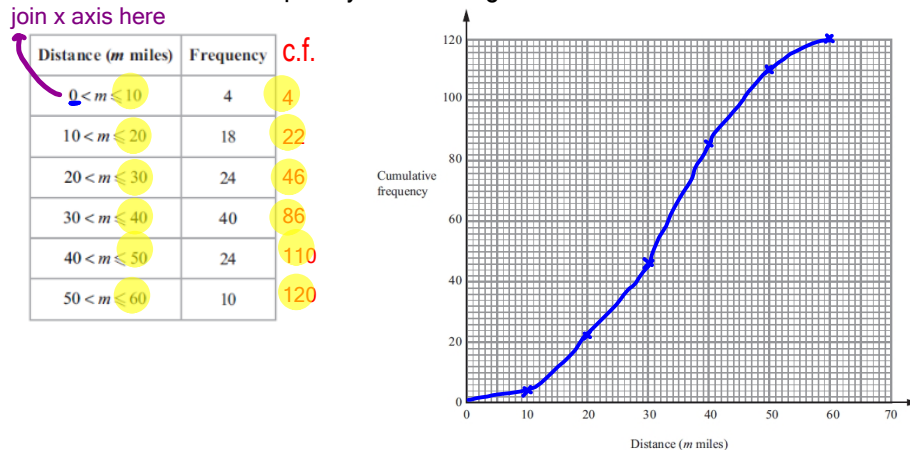
Cumulative frequency: drawing

- 1) Add a cumulative frequency column to table
- 2) Plot points on graph, using the upper value of each class
- 3) Join points up to form a smooth S shape curve (the curve starts from the lower value of the first class)

Example:

Lyndsey records the number of miles (m) she drives each day for 120 days. Some information about the results is given in the table.

Draw a cumulative frequency curve using the results.



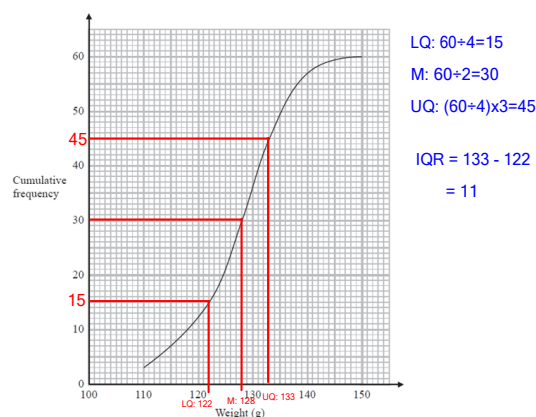
Cumulative frequency: estimating

To find Lower Quartile/Median/Upper Quartile:

- 1) Work out the following:
 - Lower Quartile: total frequency $\div 4$
 - Median: total frequency $\div 2$
 - Upper Quartile: (total frequency $\div 4$) $\times 3$
- 2) Draw a line across from this value on the cumulative frequency (y-axis)
- 3) When you hit the curve draw a line down and read off value

Example:

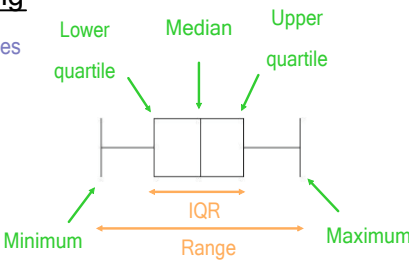
The cumulative frequency graph shows information about the weight of 60 apples. Find the Inter-Quartile-Range.



- LESS THAN: draw a line up from x-axis to hit curve and across.
The value on the y-axis is your answer.
- MORE THAN: draw a line up from x-axis to hit curve and across,
Answer is total frequency MINUS this value.

Box plots: drawing

- 1) Find all necessary values
- 2) Draw axis
- 3) Plot graph



Example:

The table shows some information about the weights, in kg, of some boxes:

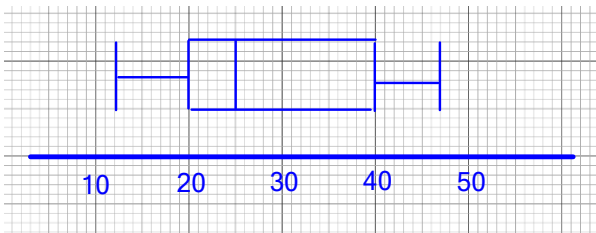
Minimum	Lower Quartile	Median	Upper Quartile	Range
12	20	25	40	47

Draw a box plot to represent this information.

$\text{Range} = \text{Maximum} - \text{Minimum}$

$35 = \text{Maximum} - 12$

$47 = \text{Maximum}$



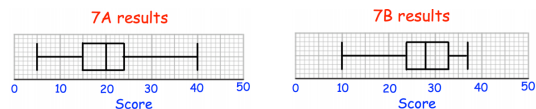
Box plots: comparing

You need to:

- Write 1 sentence to compare the medians
- Write 1 sentence to compare the IQR or range
- Make reference to the context or situation in the question in order to get full marks!

Example 1:

Compare the distributions of results in a test taken by students in 7A with the distribution of results from students in 7B

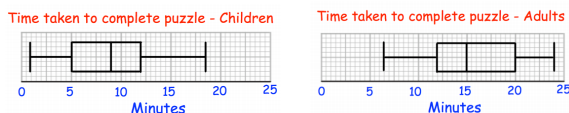


Students in 7B had a higher median, so **on average** they did better in the test than students in 7A context!

Students in 7B had a smaller range, so their data was less spread out, therefore more consistent context!

Example 2:

Compare the distributions of the times taken to complete a puzzle by children with the distribution of times by adults



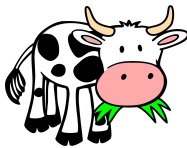
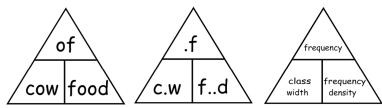
The adults had a higher median, so **on average** they took longer to complete the puzzle than the children context!

The children had a smaller IQR, so their data was less spread out, therefore more consistent context!

Histograms: drawing

- 1) Add class width and frequency density column
- 2) Identify a matching pair: row and bar
- 3) Add frequency density scale (y-axis)
- 4) Draw on missing bars and fill in table

NOTE: the area of a bar represents FREQUENCY

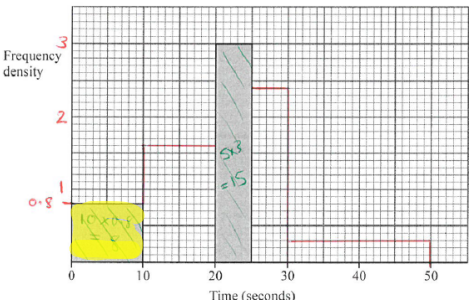


Example:

One Monday, Victoria measured the time, in seconds, that individual birds spent on her bird table.

She used this information to complete the frequency table.

Time (t seconds)	Frequency	c.w	f.d
0 < t ≤ 10	8	10	0.8
10 < t ≤ 20	16	10	1.6
20 < t ≤ 25	15	5	3.0
25 < t ≤ 30	12	5	2.4
30 < t ≤ 50	6	20	0.3

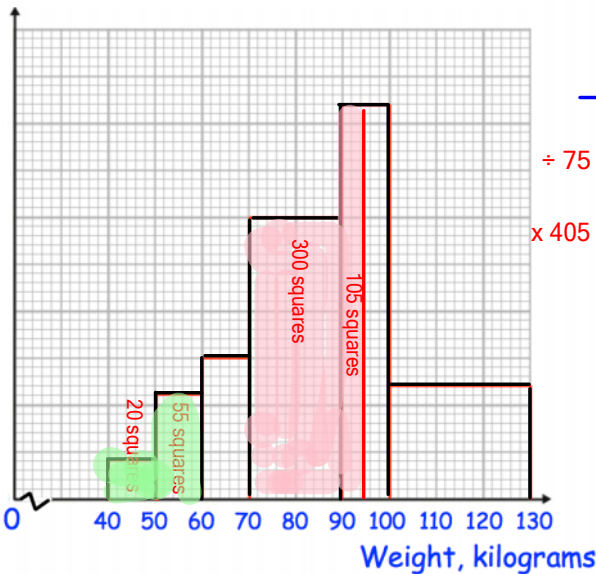


Histograms: estimating

If the histogram is drawn for you but there is **no scale** on the Frequency Density axis (and there is no table given) **use a proportion table**.

Example:

The histogram gives information about the weights of athletes.



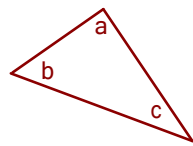
Area	Athletes
75	45
1	0.6
405	243

243 athletes weigh between 70 and 95kg.

45 athletes weigh under 60kg.
Calculate an estimate of the number of athletes between 70 and 95kg.

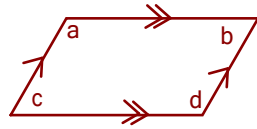
Angle facts

Triangle - all angles add up to 180°



$$a + b + c = 180$$

Parallelogram - opposite angles are equal



$$a = d \text{ and } b = c$$

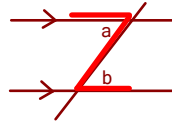
Also,

$$a + c = 180 \text{ and } b + d = 180$$

Angles in Parallel Lines:

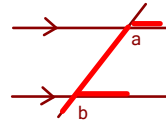
1) Alternate (Z) angles

$$a = b$$



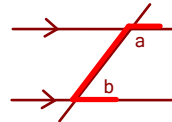
2) Corresponding (F) angles

$$a = b$$



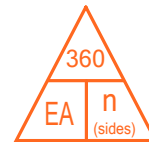
3) Co-interior (C) angles

$$a + b = 180$$



Exterior and interior angles

- To find an EXTERNAL angle in a regular polygon:



- To find an INTERNAL angle in a regular polygon:

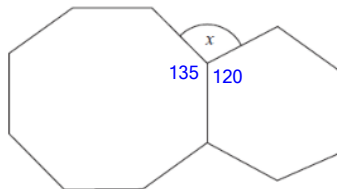
$$EA + IA = 180^\circ$$



Example:

The diagram shows a regular octagon and a regular hexagon.

Find the size of the angle marked x.



Octagon

$$EA: 360 \div 8 = 45$$

$$IA: 180 - 45 = 135$$

Hexagon

$$EA: 360 \div 6 = 60$$

$$IA: 180 - 60 = 120$$

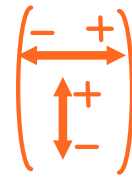
Angles around a point add up to 360° :

$$360 - (135 + 120) = 105^\circ$$

- The formula for sum of interior angles:

$$180(n - 2)$$

Transformations: translations

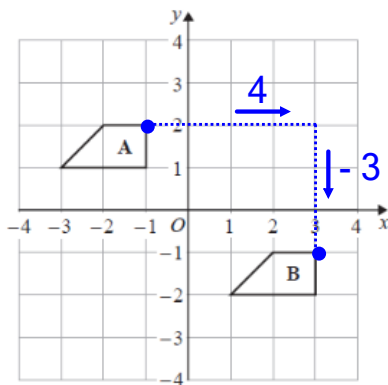


- 1) Translations can be described using column vectors
- 2) Count how to get from one point on a shape to the corresponding point on another shape

Example:

Describe the single transformation that maps shape A onto shape B.

NOTE: this means start at A and go to B



Translation vector $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ 4 right $\rightarrow$
3 down $\downarrow$

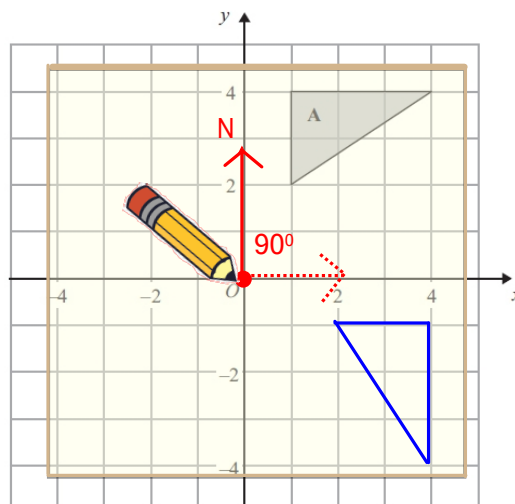
Extension: if you are combining translations then just add the vectors!

Transformations: rotations

- 1) Draw a North arrow on tracing paper at centre of rotation
- 2) Hold pencil on arrow and turn paper clockwise/anti-clockwise the desired number of degrees (90, 180 or 270)
- 3) Re-draw shape in new position

Example:

Rotate triangle 90° clockwise centre (0, 0).

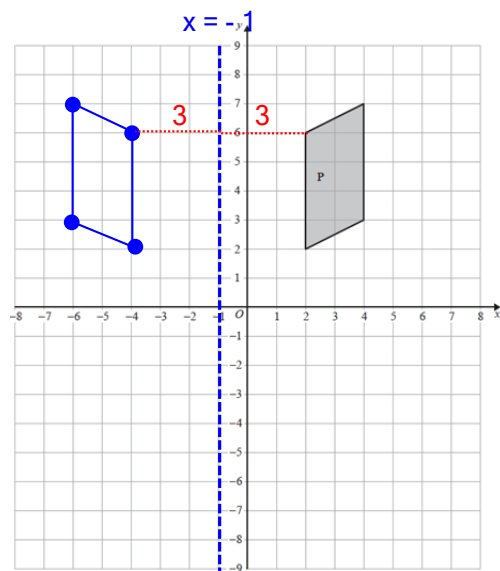


Transformations: reflections

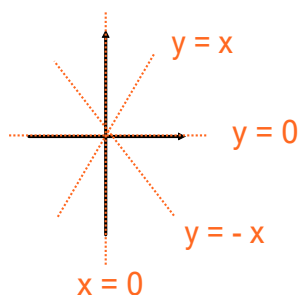
- 1) Draw on line of reflection
- 2) Count distance from each point of the shape to line of reflection
- 3) Count the same amount on the other side of the line of reflection

Example:

Reflect shape P in the line $x = -1$



Lines you should know ...

Transformations: Enlargements

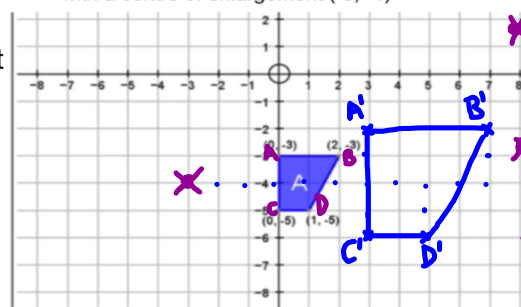
- 1) Plot the centre of enlargement on the grid
- 2) Write the column vector to each point from the centre
- 3) Multiply all the vectors by the s.f.
- 4) Map out new column vector from the centre of enlargement to find and plot points of the enlarged shape

Example:

Enlarge triangle A by scale factor 2, using centre of enlargement (5, -5)

Enlarge shape A by a scale factor of 2
with a centre of enlargement (-3, -4)

vector journey from the centre



$$\begin{aligned}
 X \rightarrow A \begin{pmatrix} 3 \\ 1 \end{pmatrix} \times 2 &= \begin{pmatrix} 6 \\ 2 \end{pmatrix} \rightarrow A' \\
 X \rightarrow B \begin{pmatrix} 5 \\ 1 \end{pmatrix} \times 2 &= \begin{pmatrix} 10 \\ 2 \end{pmatrix} \rightarrow B' \\
 X \rightarrow C \begin{pmatrix} 3 \\ -1 \end{pmatrix} \times 2 &= \begin{pmatrix} 6 \\ -2 \end{pmatrix} \rightarrow C' \\
 X \rightarrow D \begin{pmatrix} 4 \\ -1 \end{pmatrix} \times 2 &= \begin{pmatrix} 8 \\ -2 \end{pmatrix} \rightarrow D'
 \end{aligned}$$

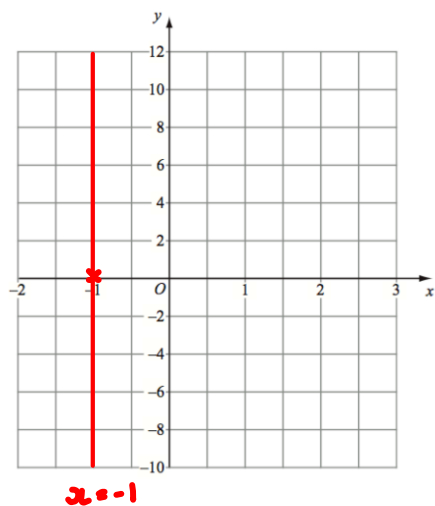
Coordinate geometry: Basic graphs

1) $x = a$ is a horizontal line

2) $y = a$ is a vertical line

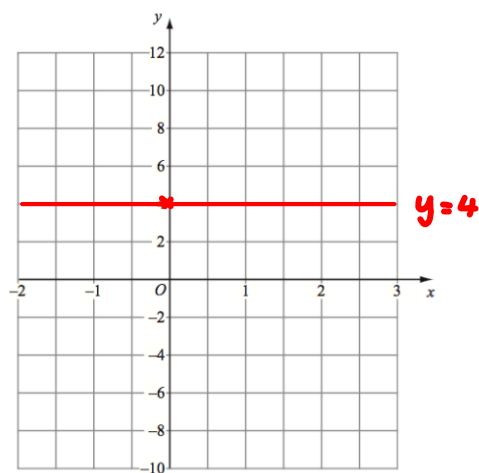
Example 1:

Draw the line $x = -1$



Example 2:

Draw the line $y = 4$

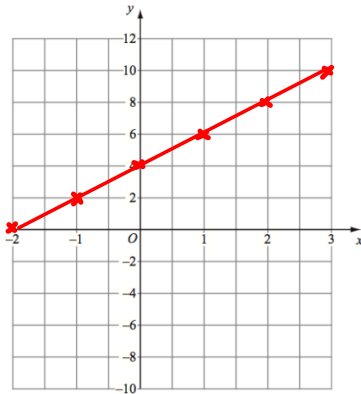


Coordinate geometry: plotting graphs

- 1) Table of values (use x-values from question)
- 2) Sub x into equation to find y-values - use () on calculator
- 3) Plot each pair as a straight line
- 4) Join with a ruler

Example 1:

Draw the line $y = 2x + 4$ for values of x from -2 to 3



x	-2	-1	0	1	2	3
y	0	2	4	6	8	10

on calculator:

$$2(\quad) + 4$$

$$2(-2) + 4 = 0$$

$$2(-1) + 4 = 2$$

$$2(0) + 4 = 4$$

$$2(1) + 4 = 6$$

$$2(2) + 4 = 8$$

$$2(3) + 4 = 10$$

Graphing linear inequalities

Approach this question one inequality at a time and do the following:

- 1) Rearrange each equation to the form $y = mx + c$
- 2) Use a table of values to plot each line on the graph
- 3) For $y >$ or $y <$ use -----.
For $y \leq$ or $y \geq$ use _____
- 4) For $y >$ or $y \geq$ hatch above the line
- 5) For $y <$ or $y \leq$ hatch below the line
- 6) The area with all hatched sides is the region you need.

Example:

On the grid below, show by shading, the region defined by the inequalities below. Mark this region with the letter R.

$$x + y < 6$$

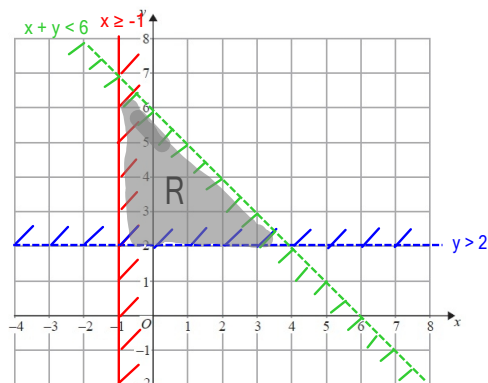
rearrange:
 $y < 6 - x$
shade below

$$x \geq -1$$

shade right

$$y \geq 2$$

shade above



Coordinate geometry: $y = mx + c$

In the form

$$y = mx + c$$

gradient y-intercept

Example 1:

Find the gradient and y-intercept of the line $y = 2x - 3$

$$\begin{aligned}\text{gradient} &= 2 \\ \text{y-intercept} &= -3\end{aligned}$$

Example 2:

Find the gradient and y-intercept of the line $y = 5 - 3x$

$$\begin{aligned}\text{gradient} &= -3 \\ \text{y-intercept} &= 5\end{aligned}$$

Example 3:

Find the gradient and y-intercept of the line $2y - 3x = 6$

$$\begin{aligned}2y - 3x &= 6 \\ + 3x &+ 3x \\ \frac{2y}{2} &= \frac{6}{2} + \frac{3x}{2} \\ y &= \frac{3}{2}x + 3 \\ \text{gradient} &= \frac{3}{2} \\ \text{y-intercept} &= 3\end{aligned}$$

needs to be rearranged
to make y the subject

Coordinate geometry: gradient

For any two points (x_1, y_1) and (x_2, y_2) , which may be given or identified from a graph of a straight line.

Gradient:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1:

Find the gradient of the line segment joining $(2, -2)$ and $(0, -6)$

$$m = \frac{-6 - (-2)}{0 - 2} = \frac{-4}{-2} = 2$$

Example 2:

The line segment joining $(5, -2)$ and $(8, c)$ has a gradient of 3.

Find the value of c .

$$3 = \frac{c - (-2)}{8 - 5}$$

$$3 = \frac{c + 2}{3}$$

$$13 \quad 13$$

$$9 = c + 2$$

$$-2 \quad -2$$

$$7 = c$$

$$c = 7$$

Coordinate geometry: Length of a line

For any two points (x_1, y_1) and (x_2, y_2) , which may be given or identified from a graph of a straight line.

Length of a line (distance between two points):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1:

Find the length of the line segment joining $(2, -2)$ and $(0, -6)$

$$\begin{aligned} d &= \sqrt{(0 - 2)^2 + (-6 - (-2))^2} \\ &= \sqrt{(-2)^2 + (-4)^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} \end{aligned}$$

Example 2:

The length of the line segment joining $(1, 2)$ and $(16, p)$ is 17.

Find the possible values of p .

$$\begin{aligned} 17 &= \sqrt{(16 - 1)^2 + (p - 2)^2} \\ 17^2 &= \sqrt{(15)^2 + (p - 2)^2} \\ 289 &= 225 + p^2 - 4p + 4 \\ 289 &= 229 + p^2 - 4p \\ -289 + 289 & \\ 0 &= p^2 - 4p - 60 \\ 0 &= (p - 10)(p + 6) \\ p &= 10 \quad p = -6 \end{aligned}$$

Coordinate geometry: Midpoint of a line

For any two points (x_1, y_1) and (x_2, y_2) , which may be given or identified from a graph of a straight line.

Midpoint of a line:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

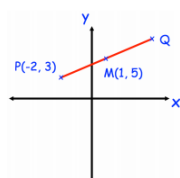
Example 1:

Find the midpoint of the line segment joining $(2, -2)$ and $(0, -6)$

$$\begin{aligned} & \left(\frac{2+0}{2}, \frac{-2+(-6)}{2} \right) \\ &= \left(\frac{2}{2}, \frac{-8}{2} \right) \\ &= (1, -4) \end{aligned}$$

Example 2:

$M(1, 5)$ is the midpoint of PQ . The coordinates of P are $(-2, 3)$, find the coordinates of Q .



$$\begin{aligned} & \left(\frac{-2+x}{2}, \frac{3+y}{2} \right) = (1, 5) \\ & \frac{-2+x}{2} = 1 \quad \frac{3+y}{2} = 5 \\ & \frac{-2+x}{2} \times 2 = 1 \times 2 \quad \frac{3+y}{2} \times 2 = 5 \times 2 \\ & -2+x = 2 \quad 3+y = 10 \\ & \begin{array}{r} -2+x = 2 \\ +2 \quad +2 \\ \hline x = 4 \end{array} \quad \begin{array}{r} 3+y = 10 \\ -3 \quad -3 \\ \hline y = 7 \end{array} \\ & Q = (4, 7) \end{aligned}$$

Coordinate geometry: Equation of a line

$$y - y_1 = m(x - x_1)$$

To use the formula you need...

- ☐ gradient
- ☐ pair of coordinates

Example 1 (given gradient and a point):

Find the equation of the line with gradient 2 that passes through the point $(4, -2)$.

$$\begin{aligned} & y - y_1 = m(x - x_1) \\ & y - (-2) = 2(x - 4) \\ & y + 2 = 2x - 8 \\ & y = 2x - 10 \end{aligned}$$

Example 2 (given two points):

Find the equation of the line that passes through the points $(5, 7)$ and $(3, -1)$.

$$m = \frac{-1-7}{3-5} = \frac{-8}{-2} = 4$$

We aren't given the gradient

$$\text{so use } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\begin{aligned} & y - 7 = 4(x - 5) \\ & y - 7 = 4x - 20 \\ & y = 4x - 13 \end{aligned}$$

You can use either pair of coordinates here

Example 3 (in the form $ax + by + c = 0$):

Find the equation of the line that passes through the points $(-5, 3)$ and $(7, 5)$. Leave your answer in the form $ax + by + c = 0$

$$m = \frac{5-3}{7-(-5)} = \frac{2}{12} = \frac{1}{6}$$

$$y - 3 = \frac{1}{6}(x + 5)$$

$$\begin{aligned} & 6y - 18 = 1(x + 5) \\ & 6y - 18 = x + 5 \end{aligned}$$

$$\begin{aligned} & -x - 6y - 18 = 5 \\ & -x - 6y - 23 = 0 \end{aligned}$$

always multiply by the denominator to clear the fraction before you expand the brackets!

Coordinate geometry: parallel and perpendicular lines

If $m = a$

Gradient of parallel line = a

Gradient of perpendicular line = $-\frac{1}{a}$

$$y - y_1 = m(x - x_1)$$

To use the formula you need...

□ gradient

□ pair of coordinates

Example 1:

$$y = -5x + 3$$

a) Write down the equation of a line that is parallel

b) Write down the equation of a line that is perpendicular

$$m = -5$$

$$a) y = -5x + 6$$

$$b) y = \frac{1}{5}x - 5$$

The + c can take any value as the question asks for the equation of any line that is parallel or perpendicular.

Example 2:

Find the equation of the line that is parallel to $y = 2x - 1$ and passes through the point $(1, 8)$.

$$m = 2$$

$$y - 8 = 2(x - 1)$$

$$y - 8 = 2x - 2$$

$$y = 2x + 6$$

Use $y - y_1 = m(x - x_1)$ to find the equation of a line that goes through a specific point.

Example 3:

Find the equation of the line that is perpendicular to $x + 4y - 6 = 0$ and passes through the point $(1, 8)$.

$$x + 4y - 6 = 0$$

$$-x$$

$$4y - 6 = -x$$

$$\frac{4y}{4} = \frac{-x}{4} + \frac{6}{4}$$

$$y = -\frac{1}{4}x + \frac{3}{2}$$

$$m = -\frac{1}{4}$$

$$b. m = 4$$

$$y - 8 = 4(x - 1)$$

$$y - 8 = 4x - 4$$

$$y = 4x + 4$$

Need to rearrange into $y = mx + c$ form to identify the gradient.

Perpendicular gradient is negative reciprocal

Coordinate geometry: finding the equation from the graph

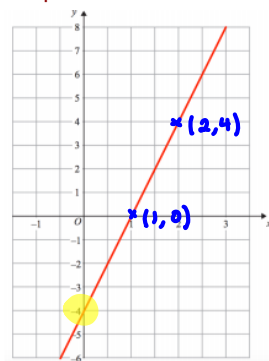
1) horizontal line: $x = a$

2) vertical line: $y = a$

3) For all other lines:

- choose two points and use the gradient formula
- locate y-intercept
- sub into $y = mx + c$

Example 1:

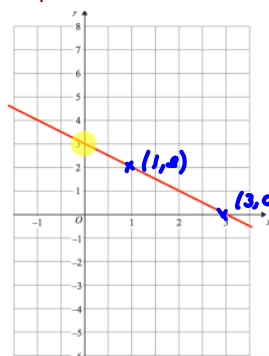


$$m = \frac{4 - 0}{2 - 1} = \frac{4}{1} = 4$$

$$y\text{-intercept} = -4$$

$$y = 4x - 4$$

Example 2:



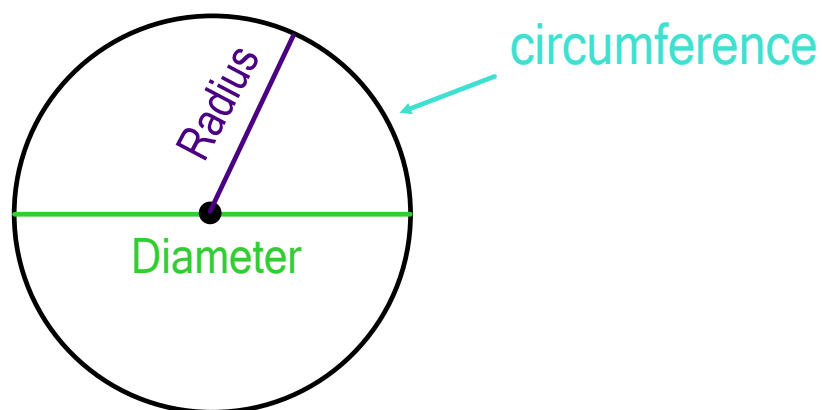
$$m = \frac{0 - 0}{1 - 3} = \frac{0}{-2} = 0$$

$$y\text{-intercept} = 3$$

$$y = -x + 3$$

Equation of a circle

$$x^2 + y^2 = r^2 \quad \text{where centre is } (0, 0)$$



Equation of tangent to a circle

1) Find gradient of radius from centre to point

Gradient: $\frac{y_2 - y_1}{x_2 - x_1}$

2) Find perpendicular gradient ($-1 \div$ ANS)

3) Sub values into $y - y_1 = m(x - x_1)$

Example:

Note: centre of the circle is always (0, 0)

T is a circle with equation $x^2 + y^2 = 9$

P(5, 2) is a point on T.

Find an equation of the tangent to T at the point P.

Gradient of radius:

$$\frac{2 - 0}{5 - 0} = \frac{2}{5}$$

Gradient of tangent:

$$-1 \div \frac{2}{5} = \frac{-5}{2}$$

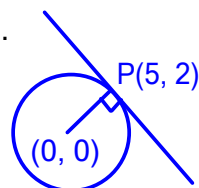
Equation of tangent:

$$y - y_1 = m(x - x_1) \quad (5, 2)$$

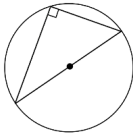
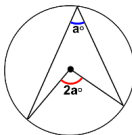
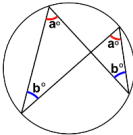
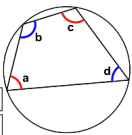
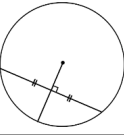
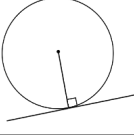
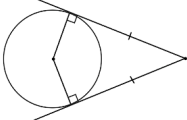
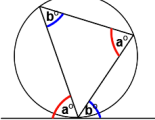
$$y - 2 = -\frac{5}{2}(x - 5)$$

$$y - 2 = -\frac{5}{2}x + \frac{25}{2}$$

$$y = -\frac{5}{2}x + \frac{29}{2}$$

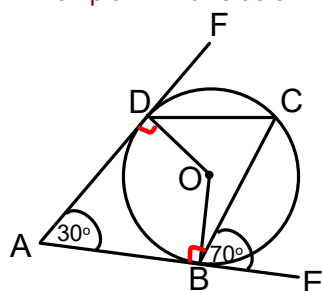


CIRCLE THEOREMS

Circle Theorems: Angles in Circles	
The angle at the circumference in a semi-circle is a right angle (90°).	The angle at the centre is twice the angle at the circumference.
	
Angles in the same segment are equal.	Opposite angles in a cyclic quadrilateral add up to 180°.
	 $a + c = 180$ $b + d = 180$
Circle Theorems: Tangents and Circles	
The line from the centre of a circle through a chord is a perpendicular bisector.	The angle between a tangent and the radius is a right angle (90°).
	
Two tangents drawn from the same point to a circle are equal in length.	Alternate Segment Theorem: The angle between a tangent and a chord is equal to the angle at the circumference in the alternate segment.
	

CIRCLE THEOREMS

Example 1: Find value of missing angle



B , C and D are points on the circumference of a circle, centre O .
 ABE and ADF are tangents to the circle.
 Angle $DAB = 30^\circ$
 Angle $CBE = 70^\circ$

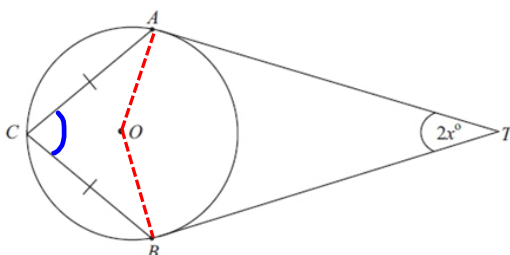
Work out the size of angle ODC .
 Give a reason for each stage in your working.

Maths	Reason
$\angle ODA = \angle OBA = 90$	angle between radius and tangent is 90°
$\angle DOB = 360 - 90 - 90 - 30 = 150$	angles in a quadrilateral add up to 360°
$\angle DCB = \frac{150}{2} = 75$	angle at the centre is twice the angle at the circumference
reflex $\angle DOB = 360 - 150 = 210$	angles around a point add up to 360°
$\angle OBC = 90 - 70 = 20$	angle between radius and tangent is 90°
$\angle ODC = 360 - 75 - 20 - 210 = 55$	$DOBC$ is a quadrilateral, angles in a quadrilateral add up to 360°

Answer = 55°

CIRCLE THEOREMS

Example 2: Find expression for missing angle

Diagram **NOT** accurately drawn

A , B and C are points on the circumference of the circle, centre O .
 TA and TB are tangents to the circle.

 $CA = CB$.Angle $ATB = 2x^\circ$.Find an expression for angle ACB in terms of x .

Maths	Reason
$\angle OAT = \angle OBT = 90$	angle between radius and tangent is 90°
$\angle AOB = 360 - 90 - 90 - 2x$ $= 180 - 2x$	angles in a quadrilateral add up to 360°
$\angle ACB = \frac{180 - 2x}{2}$ $= 90 - x$	angle at the centre is twice the angle at the circumference

Graph Transformations

Outside Bracket $\Rightarrow$ changes $y \Rightarrow$ expected ✓

Inside Bracket $\Rightarrow$ changes $x \Rightarrow$ opposite ✗

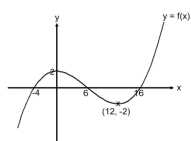
$f(x + a)$	$(x - a, y)$
$f(x) + a$	$(x, y + a)$
$f(ax)$	$(x/a, y)$
$af(x)$	(x, ay)

Special Cases (learn!):

$y = f(-x) \Rightarrow$ reflection in y -axis

$y = -f(x) \Rightarrow$ reflection in x -axis

Example:



Sketch the graph of the following:

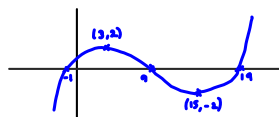
a) $y = f(x - 3)$ inside - changes x - opposite

b) $y = 2f(x)$ outside - changes y - expected

c) $y = -f(x)$ special case - reflection in x -axis

a) $(x, y) \rightarrow (x+3, y)$

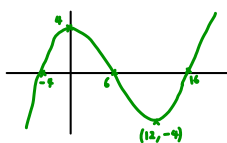
$(-4, 0)$	$(-1, 0)$
$(0, 2)$	$(3, 2)$
$(6, -2)$	$(9, -2)$
$(12, 2)$	$(15, 2)$
$(16, 0)$	$(19, 0)$



The shape of the graph will stay the same.

b) $(x, y) \rightarrow (x, 2y)$

$(-4, 0)$	$(-4, 0)$
$(0, 2)$	$(0, 4)$
$(6, -2)$	$(6, -4)$
$(12, 2)$	$(12, 4)$
$(16, 0)$	$(16, 0)$

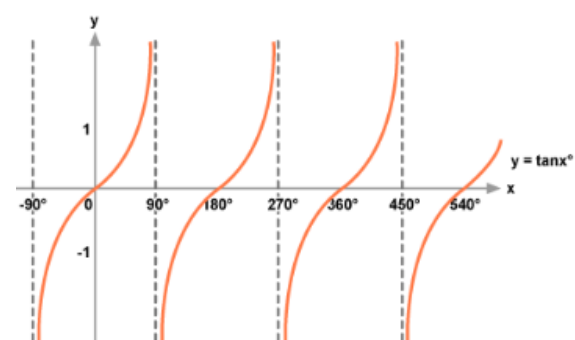
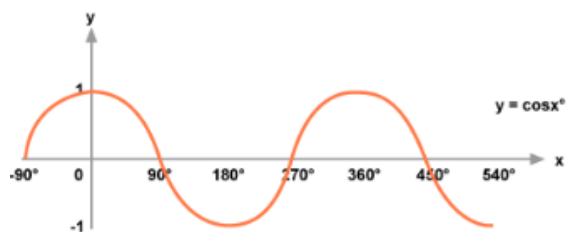
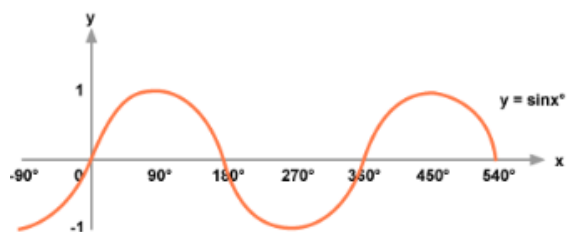


c) $(x, y) \rightarrow (x, -y)$

$(-4, 0)$	$(-4, 0)$
$(0, 2)$	$(0, -2)$
$(6, -2)$	$(6, 2)$
$(12, 2)$	$(12, -2)$
$(16, 0)$	$(16, 0)$



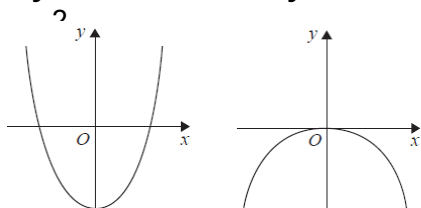
Trigonometric graphs



Types of graphs

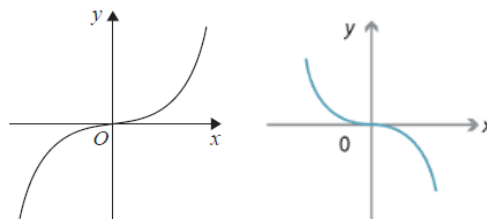
Quadratic

$$y = x^2 \quad \text{or} \quad y = -x^2$$



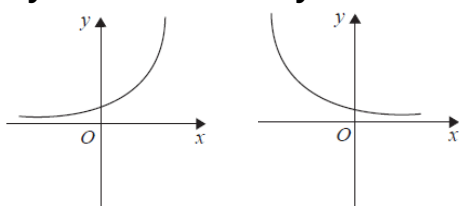
Cubic

$$y = x^3 \quad \text{or} \quad y = -x^3$$



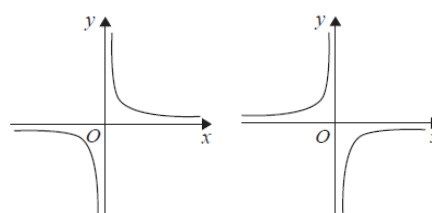
Exponential

$$y = a^x \quad \text{or} \quad y = -a^x$$



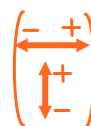
Reciprocal

$$y = \frac{a}{x} \quad \text{or} \quad y = -\frac{a}{x}$$



Column vectors

- Column vectors are used to describe translations
- You may need to do calculations with them too!



Example 1: $a = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $b = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

a) Write down $a + b$ as a column vector

$$\begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 + 5 \\ 3 + -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

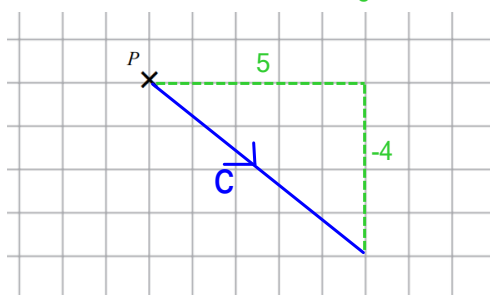
b) Write down $2a - b$ as a column vector

$$2 \begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 - 5 \\ 6 - -1 \end{pmatrix} = \begin{pmatrix} -9 \\ 7 \end{pmatrix}$$

Example 2:

Given that $c = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$, from the point P, draw the vector c

5 right and 4 down



Remember to
label the direction
too!

Vectors

1) Write down the route you can take e.g. $\vec{AD} = \vec{AB} + \vec{BD}$

2) Use vectors to describe the journey

3) Add any new vectors to your diagram

NOTE: vectors are the same if they are parallel, equal in length and go in the same direction (if they face the other direction add a minus in front)

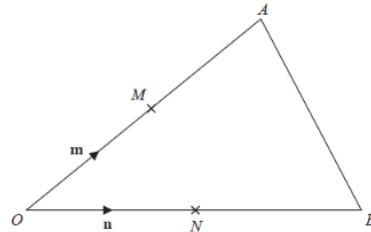
Example:

OAB is a triangle.

M is the midpoint of OA.

N is the midpoint of OB.

Show that AB is parallel to MN.



To show vectors are parallel (or lie on a straight line) you must show that one is a multiple of the other.

To find $\vec{AB}$:

$$\vec{AB} = \vec{AO} + \vec{OB}$$

$$\vec{AB} = -\vec{m} + \vec{n}$$

To find $\vec{MN}$:

$$\vec{MN} = \vec{MO} + \vec{ON}$$

$$\vec{MN} = -0.5\vec{m} + 0.5\vec{n}$$

$$\vec{MN} = 0.5(-\vec{m} + \vec{n})$$

$$\vec{MN} = 0.5(\vec{AB})$$

MN is a multiple of AB
parallel

Congruent triangles

When two triangles are congruent they will have one of the following properties:

1) Three sides are the same (SSS rule):

The three sides of the first triangle are equal to the three sides of the second triangle



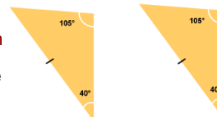
2) Two sides and one angle are the same (SAS rule):

Two sides of the first triangle and the included angle are equal to two sides of the second triangle and the included angle is equal.



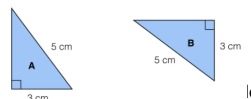
3) Two angles and one side are the same (ASA rule):

Two angles in the first triangle are equal to two angles of the second triangle and one corresponding side is equal.



4) Right-angle, hypotenuse and side (RHS rule):

In two right-angled triangles, the length of one side, which is not the hypotenuse, is equal to the length of the hypotenuse and corresponding side of the other triangle



Bearings

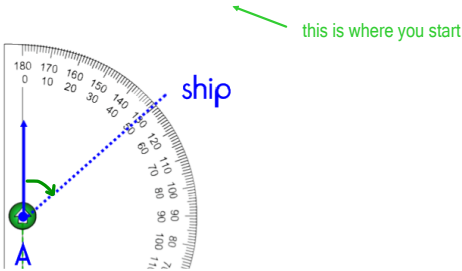
- 1) Measure clockwise
- 2) Measure from direction of North
- 3) Write as a 3 digit value



NOTE: always draw a diagram!

Example 1: Measuring a bearing

A ship is on a bearing of 050° from A.

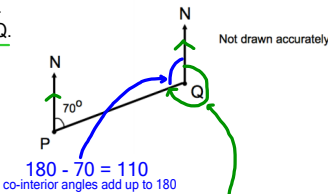


Example 2: Calculating a bearing

The diagram shows the position of two airplanes, P and Q.
The bearing of Q from P is 070°.
Calculate the bearing of P from Q.

this is where you start

The North lines are parallel, so use the facts you know about angles on parallel lines

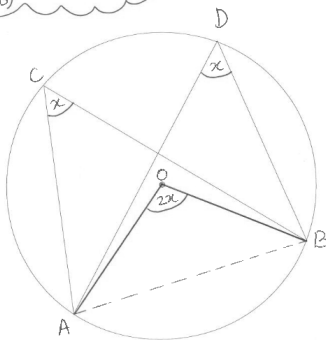


Bearing of P from Q = 360 - 110 = 250
angles around a point add up to 360

Answer = 250°

Circle theorem proofs

- Label centre and points on circumference
- Draw two radii from centre (OA and OB)

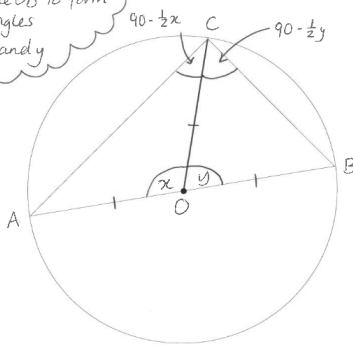


Prove that angles in the same segment are equal

Maths	Reason
Let $\angle ACB = x$	
$\angle AOB = 2 \times x = 2x$	Angle at centre is twice the angle at the circumference
$\angle ADB = 2x \div 2 = x$	Angle at circumference is half the angle at the centre.
	$\angle ACB = \angle ADB$ which are both angles in the same segment.

Circle theorem proofs

- Label centre and points at circumference
- Draw line OB to form two triangles
- Label x and y

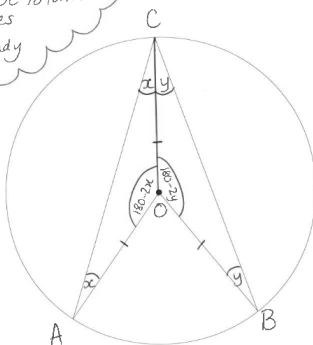


Prove the angle subtended at the circumference by a semicircle is a right angle

Maths	Reason
Let $\angle AOC = x$ and $\angle BOC = y$	
$x + y = 180$	Angles on a straight line add to 180° .
$\angle OAC = \angle OCA = \frac{180 - x}{2}$ $= 90 - \frac{1}{2}x$	Base angles of an isosceles are equal Angles in a triangle add up to 180°
Similarly, $\angle OBC = \angle OCB = \frac{180 - y}{2}$ $= 90 - \frac{1}{2}y$	Base angles of an isosceles are equal Angles in a triangle add up to 180° .
$\angle ACB = 90 - \frac{1}{2}x + 90 - \frac{1}{2}y$ $= 180 - \frac{1}{2}x - \frac{1}{2}y$ $= 180 - \frac{1}{2}(x + y)$ [since $x + y = 180$ from above] $= 180 - \frac{1}{2}(180)$ $= 90^\circ$	$\angle ACB = 90^\circ$ which is the angle subtended at the circumference by the semi-circle

Circle theorem proofs

- Label centre and points at circumference
- Draw line OC to form two triangles
- Label x and y

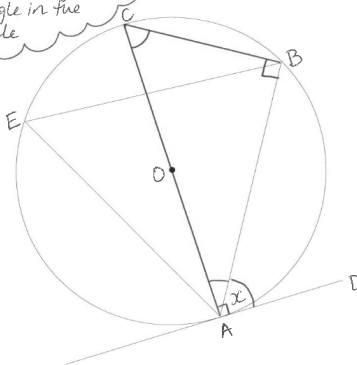


Prove that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference

Maths	Reason
Let $\angle OCA = x$ and $\angle OCB = y$ $\angle OCA = \angle OAC = x$	Base angles of an isosceles are equal (OC and OA are both radius)
$\angle COA = 180 - 2x$	Angles in a triangle add up to 180°
Similarly, $\angle OCB = \angle OBC = y$	Base angles of an isosceles are equal
$\angle COB = 180 - 2y$	Angles in a triangle add up to 180°
$\angle AOB = 360 - (180 - 2x) - (180 - 2y)$ $= 360 - 180 + 2x - 180 + 2y$ $= 2x + 2y$ $= 2(x + y)$ $= 2(\angle ABC)$	$\angle AOB$ is the angle at the centre, which is twice $\angle ABC$ which is the angle at the circumference.

Circle theorem proofs

- Label centre and points at circumference
- Draw diameter and chord to form a right angle triangle in the semi-circle

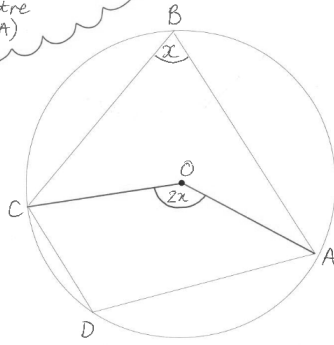


Prove the alternate segment theorem

Maths	Reason
Let $\angle BAD = x$	
$\angle BAC = 90 - x$	Angle where tangent meets radius is 90°
$\angle ACB = 180 - 90 - (90 - x)$ $= 180 - 90 - 90 + x$ $= x$	Angle at circumference in a semi-circle is 90° AND Angles in a triangle add up to 180°
$\angle AEB = x$	Angles in the same segment are equal
	$\angle BAD = \angle AEB = x$ where $\angle AEB$ is in the alternate segment to $\angle BAD$.

Circle theorem proofs

- Label centre and points at circumference
- Draw two radii from centre (OC and OA)



Prove that opposite angles of a cyclic quadrilateral sum to 180°

Maths	Reason
Let $\angle CBA = x$	Angle at centre is twice the angle at the circumference.
So $\angle COA = 2x$	
reflex $\angle COA = 360 - 2x$	Angles around a point add to 360°
$\angle CDA = \frac{360 - 2x}{2} = 180 - x$	Angle at circumference is half the angle at the centre.
$\angle CBA + \angle CDA$	
$= x + 180 - x$	$\angle CBA + \angle CDA = 180$ which are opposite angles of a cyclic quadrilateral.
$= 180^\circ$	

Edexcel GCSE (9-1) Maths: need-to-know formulae

www.edexcel.com/gcsemathsformulae

Areas

Rectangle = $l \times w$

Parallelogram = $b \times h$

Triangle = $\frac{1}{2} b \times h$

Trapezium = $\frac{1}{2} (a + b) h$

Circles

Circumference = $\pi \times \text{diameter}$, $C = \pi d$

Circumference = $2 \times \pi \times \text{radius}$, $C = 2\pi r$

Area of a circle = $\pi \times \text{radius squared}$, $A = \pi r^2$

Pythagoras

Pythagoras' Theorem

For a right-angled triangle,

$a^2 + b^2 = c^2$

Trigonometric ratios (new to F)

$\sin x^\circ = \frac{\text{opp}}{\text{hyp}}$, $\cos x^\circ = \frac{\text{adj}}{\text{hyp}}$, $\tan x^\circ = \frac{\text{opp}}{\text{adj}}$

Quadratic equations

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Volumes

Cuboid = $l \times w \times h$

Prism = area of cross section $\times$ length

Cylinder = $\pi r^2 h$

Pyramid = $\frac{1}{3} \times \text{area of base} \times h$

Compound measures

Speed

speed = $\frac{\text{distance}}{\text{time}}$

Density

density = $\frac{\text{mass}}{\text{volume}}$

Pressure

pressure = $\frac{\text{force}}{\text{area}}$

Trigonometric formulae

Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$

Foundation tier formulae

Higher tier formulae

other useful formulae:

other useful formulae:

key notes:

key notes:

key notes: